经济波动

Ramsey v.s. RBC → DNK

	Ramsey	RBC
内生 variables	$K(t)$, $C(t)$	N
外生 variables	N, A	$K=1$, $A \sim AR(1)$

竞争性市场

RBC model

家庭

统计意义上期望

更重视当前效用
主观(时间)贴现因子

$$\boxed{\begin{aligned} &\max\ E_0 \sum_{t=0}^{\infty} \beta^t u(C_t, N_t) \qquad \beta \in (0,1) \\ &\text{s.t.}\quad P_t C_t + Q_t B_t \le \underbrace{B_{t-1}}_{\text{债券(资产所得)}} + \underbrace{W_t N_t}_{\text{劳动收入}} + D_t \end{aligned}}$$

(支出 ≤ 收入)

D_t：外生 variable (类似扰动项)
厂商利润=0 / 政府税收 / 转移支付

$$Q_t = \frac{1}{1+i_t}$$ 贴现到第t期初

i_t：nominal interest rate

求解：$\mathcal{L} = E_0 \sum_{t=0}^{\infty} \beta^t \left[u(C_t, N_t) + \lambda_t (B_{t-1} + W_t N_t + D_t - P_t C_t - Q_t B_t) \right]$

选择 variables：$\{ C_t, N_t, B_t \}$

$$\frac{\partial \mathcal{L}}{\partial C_t} = 0 \Rightarrow \beta^t (u'_{C_t} - \lambda_t P_t) = 0 \Rightarrow \frac{u'_{C_t}}{E_t u_{C_t}} = \frac{\lambda_t}{E_t \lambda_{t+1}} \frac{P_t}{E_t P_{t+1}}$$

负效用　边际替代率

$$\frac{\partial \mathcal{L}}{\partial N_t} = 0 \Rightarrow \beta^t (u'_{N_t} + \lambda_t W_t) = 0 \Rightarrow \frac{W_t}{P_t} = \frac{-u_{N_t}}{u_{C_t}}$$ 劳动供给方程

实际工资

$$\frac{\partial \mathcal{L}}{\partial B_t} = 0 \Rightarrow -\lambda_t Q_t \beta^t + E_t \beta^{t+1} \lambda_{t+1} = 0 \Rightarrow \frac{\beta E_t \lambda_{t+1}}{\lambda_t} = Q_t$$

在第t期，(t+1)期信息未知

所有下标都往前进一期 $B_{t-1} \to B_t$

$$\Rightarrow Q_t = \beta E_t \frac{u_{C_{t+1}}}{u_{C_t}} \frac{P_t}{P_{t+1}}$$ 消费 Euler Equation

Q_t：边际成本　$\beta E_t \frac{u_{C_{t+1}}}{u_{C_t}} \frac{P_t}{P_{t+1}}$：边际收益

$$u(C_t, N_t) = \begin{cases} \dfrac{C_t^{1-\sigma} - 1}{1-\sigma} - \dfrac{N_t^{1+\varphi}}{1+\varphi}, & \sigma \ne 1 \\ \ln C_t - \dfrac{N_t^{1+\varphi}}{1+\varphi}, & \sigma = 1 \end{cases}$$

σ：跨期替代弹性

$$\begin{cases} \dfrac{W_t}{P_t} = C_t^{\sigma} N_t^{\varphi} & \text{劳动供给方程} \\ Q_t = \beta E_t \left[\left(\dfrac{C_{t+1}}{C_t} \right)^{-\sigma} \dfrac{P_t}{P_{t+1}} \right] & \text{跨期消费 Euler Equation} \end{cases}$$

厂商

$$\boxed{\begin{aligned} &\max \ P_t Y_t - W_t N_t \\ &Y_t = A_t N_t^{1-\alpha} \\ &A_t = A_{t-1}^{\rho_a} e^{\varepsilon_t^a} \end{aligned}}$$ 短期利润函数

选择 variable: N_t

$\Rightarrow$ FOC: $\frac{W_t}{P_t} = (1-\alpha) A_t N_t^{-\alpha}$ 劳动需求方程

☆考 古典模型

AS
$$\begin{cases} Y = F(\overset{A=1}{K}, N) \\ \frac{W}{P} = F_N \\ N = N(\frac{W}{P}) \end{cases}$$

AD
$$\begin{cases} Y = C + I + G \\ C = C(Y_D, r-\pi) \\ I = I(q-1) \\ \frac{M}{P} = m(Y, r) \end{cases}$$

from K

i: nominal interest rate

$\downarrow$

Equilibrium?

$\downarrow$ 全微分

比较静态分析

RBC model

AS
$$\begin{cases} Y_t = \overset{K=1}{A_t} N_t^{1-\alpha} \Leftrightarrow Y = F(A_t, N_t) \\ \frac{W_t}{P_t} = F_{N_t} = (1-\alpha) A_t N_t^{-\alpha} \\ \frac{W_t}{P_t} = \frac{-U_{N_t}}{U_{C_t}} = C_t^{\sigma} N_t^{\varphi} \end{cases}$$

AD
$$\begin{cases} Y_t = C_t \\ Q_t = \beta E_t\left[\left(\frac{C_{t+1}}{C_t}\right)^{-\sigma} \frac{P_t}{P_{t+1}}\right] \\ \frac{M_t}{P_t} = \frac{Y_t}{Q_t^{\eta}} \quad (Q_t = \frac{1}{1+i_t}). \end{cases}$$

$\downarrow$ 对数线性化

Equilbrium?

$\downarrow$ 均衡路径 → 稳态

比较动态分析

对数线性化

① 定义法

Def. $\boxed{\hat{x}_t = \ln X_t - \ln X}$ 稳态值 $= x_t - x$ [偏离稳态的变化]

$= \ln \frac{X_t}{X} = \ln\left(1 + \frac{X_t - X}{X}\right) \approx \ln 1 + \frac{X_t - X}{X} = \frac{X_t - X}{X}$

② $\frac{X_t}{X} = 1 + \hat{x}_t \Rightarrow X_t = X(1 + \hat{x}_t)$

eg. $Y_t = C_t$ ($Y = C$ 稳态)

$\Rightarrow$ Def法 $\begin{cases} \ln Y_t = \ln C_t \\ \ln Y = \ln C \end{cases} \Rightarrow \hat{y}_t = \hat{c}_t$

$Y_t = C_t + I_t + G_t$ ($Y = C + I + G$)

$Y(1+\hat{y}_t) = C(1+\hat{c}_t) + I(1+\hat{i}_t) + G(1+\hat{g}_t)$

$\cancel{Y} + Y\hat{y}_t = \cancel{C} + C\hat{c}_t + \cancel{I} + I\hat{i}_t + \cancel{G} + G\hat{g}_t$

$\hat{y}_t = \frac{C}{Y}\hat{c}_t + \frac{I}{Y}\hat{i}_t + \frac{G}{Y}\hat{g}_t$

[1] $\hat{y}_t = a_t + (1-\alpha)\hat{n}_t$ 外生

[2] $\hat{w}_t - \hat{p}_t = a_t - \alpha \hat{n}_t$

[3] $\hat{w}_t - \hat{p}_t = \sigma \hat{c}_t + \varphi \hat{n}_t$

[4] $\hat{y}_t = \hat{c}_t$

难 [5] $\hat{c}_t = E_t \hat{c}_{t+1} - \frac{1}{\sigma}(\hat{i}_t - E_t \hat{\pi}_{t+1})$

$$1 = \beta E_t Q_t^{-1}\left[\left(\frac{C_{t+1}}{C_t}\right)^{-\sigma}\frac{P_t}{P_{t+1}}\right] \qquad \ln Q_t^{-1} = \ln(1+i_t) \to i_t$$

$$1 = E_t \exp\{\ln\beta - \ln Q_t - \sigma(C_{t+1} - C_t) - (p_{t+1} - p_t)\} \qquad e^{\ln x} = x$$

$$\Delta C_{t+1} \qquad \Delta p_{t+1} = \pi_{t+1}$$

$$\overset{\text{Taylor展开}}{=} E_t\left[1 + (i_t - i) - \sigma(\Delta C_{t+1} - \Delta C) - (\pi_{t+1} - \pi)\right] \qquad \text{多变量一阶Taylor展开}$$

$$= E_t \Delta \hat{i}_{t+1} = \frac{1}{\sigma}(\hat{i}_t - E_t \hat{\pi}_{t+1})$$

[6] $m_t - \hat{p}_t = \hat{y}_t - \eta \hat{i}_t \qquad (\text{def } \ln Q_t - \ln Q = \hat{i}_t)$

[7] $a_t = \rho_a a_{t-1} + \varepsilon_t^a$ AR(1)

$$A_t = A_{t-1}^{\rho_a} e^{\varepsilon_t^a} \qquad \underset{\ln A}{\ln A_t} = \rho_a \underset{\ln A}{\ln A_{t-1}} + \underset{0}{\varepsilon_t^a} \sim N(0,\sigma^2)$$

White Noise

Equilbrium

[2][3] plus [4] $\quad a_t - \alpha \hat{n}_t = \sigma \hat{c}_t + \varphi \hat{n}_t$

$$\Rightarrow \hat{n}_t = \frac{1-\sigma}{\sigma(1-\alpha)+\alpha+\varphi} a_t \qquad [8]$$

[8][1] $\quad \Rightarrow \hat{y}_t = \frac{1+\varphi}{\sigma(1-\alpha)+\alpha+\varphi} a_t = \hat{c}_t \qquad [9]$

$$\text{Def } \hat{\omega}_t = \hat{w}_t - \hat{p}_t$$

[8][2] $\quad \Rightarrow \hat{\omega}_t = \frac{\sigma+\varphi}{\sigma(1-\alpha)+\alpha+\varphi} a_t$

$$\text{Def } \hat{r}_t = \hat{i}_t - E_t \pi_{t+1}$$

$$= \sigma E_t \Delta \hat{C}_{t+1} = \sigma E_t \underbrace{\Delta \hat{y}_{t+1}}_{E_t \hat{y}_{t+1} - \hat{y}_t} = \underbrace{\sigma \frac{1+\varphi}{\sigma(1-\alpha)+\alpha+\varphi}}_{B}(E_t a_{t+1} - a_t)$$

[5][7][9] $\longrightarrow \; = B\left[E_t(\rho_a a_t + \varepsilon_{t+1}^a) - a_t\right]$

$$= B E_t\left[(\rho_a - 1)a_t + \varepsilon_{t+1}^a\right]$$

$= B(\rho_a - 1) a_t$ $\quad \swarrow E_t \varepsilon^a_{t+1} = 0$ (AR(1)) 均值为0

所有实际变量求得出，与 $m(t)$ 无关（货币供应量） ⇒ 货币中性

* RBC 货币中性的结论不绝对
if $m(t) \to U(\cdot)$

* RBC + 人口增长率 + 技术增长 类似 Ramsey → 人均有效
确保稳态的存在（消除趋势的影响）