陈书琛

DNK 企业部门最优化

1. 利润最大化 求最优定价 （需体现垄断竞争和价格粘性）

$P \times C - TC$

消费 总成本

预期算子　书上是 θ，这里用 $1-\theta$ 表现价格粘性 便于后续构造和计算

$$\max_{P_{i0}^*} \Pi_0^n = E_0 \sum_{t=0}^{\infty} Q_{0,t} (1-\theta)^t \left[P_{i0}^* \cdot Y_{i,t|0} - TC^n_{i,t|0}(Y_{i,t|0}) \right]$$

体现垄断竞争

其它类型会以产量作为选择变量

i 指的是 i 厂商调价格后不变

利润贴现

（选择变量）调整后的定价

调整后的产出 且调整后不变（粘性）（$Y_t = C_t$ 市场出清）

总成本是产出 $Y_{i,t|0}$ 的函数

$t|0$ 表示第 0 期调整定价

t 表示之后的阶段

（书上是 $t+k|t$，从第 t 期开始，没啥区别）

s.t. 目标函数： $Y_{i,t|0} = \left(\frac{P_{i0}}{P_t}\right)^{-\varepsilon} Y_t$　（由家庭部门得到 $C_{it} = \left(\frac{P_{it}}{P_t}\right)^{-\varepsilon} C_t$）

用代入法求解（将目标函数代入 max…）

得 $$\Pi_0^n = E_0 \sum_{t=0}^{\infty} Q_{0,t} (1-\theta)^t \left(P_{i0}^* \cdot \left[\left(\frac{P_{i0}^*}{P_t}\right)^{-\varepsilon} Y_t \right] - TC^n_{i,t|0}\left[\left(\frac{P_{i0}^*}{P_t}\right)^{-\varepsilon} Y_t \right] \right)$$

$P_{i0}^{*\,1-\varepsilon}$

FOC： $\frac{\partial \pi_0^n}{\partial P_{i0}^*} = 0$

$MC^n_{i,t|0}$　　$-\varepsilon \cdot \frac{P_{i0}^{*\,-\varepsilon-1}}{P_t^{-\varepsilon-1}} \cdot \frac{1}{P_t} \cdot Y_t$

$$\Rightarrow E_0 \sum_{t=0}^{\infty} Q_{0,t} (1-\theta)^t \left[(1-\varepsilon) \underbrace{\left(\frac{P_{i0}^*}{P_t}\right)^{-\varepsilon} Y_t}_{Y_{i,t|0}} - \frac{\partial TC^n_{i,t|0}}{\partial Y_{i,t|0}} \frac{\partial Y_{i,t|0}}{\partial P_{i0}^*} \right] = 0$$

$$\Rightarrow E_0 \sum_{t=0}^{\infty} Q_{0,t} (1-\theta)^t \left[(1-\varepsilon) Y_{i,t|0} + MC^n_{i,t|0} \cdot \varepsilon \underbrace{\left(\frac{P_{i0}^*}{P_t}\right)^{-\varepsilon-1} \cdot \frac{Y_t}{P_t}}_{Y_{i,t|0}} \right] = 0$$

$$\Rightarrow E_0 \sum_{t=0}^{\infty} Q_{0,t} (1-\theta)^t \left[(1-\varepsilon) Y_{i,t|0} + \frac{\varepsilon}{\cancel{P_t}} MC^n_{i,t|0} \cdot Y_{i,t|0} \cdot \left(\frac{P_{i0}^*}{\cancel{P_t}}\right)^{-1} \right] = 0$$

$$\Rightarrow E_0 \sum_{t=0}^{\infty} Q_{0,t} (1-\theta)^t \cdot Y_{i,t|0} \left[(1-\varepsilon) + \varepsilon \cdot MC^n_{i,t|0} \cdot P_{i0}^{*\,-1} \right] = 0$$

接下来展开移项

$$\Rightarrow (1-\varepsilon) E_0 \sum_{t=0}^{\infty} Q_{0,t} (1-\theta)^t \cdot Y_{i,t|0} = -\varepsilon \cdot P_{i0}^{*\,-1} \cdot E_0 \sum_{t=0}^{\infty} Q_{0,t} (1-\theta)^t \cdot Y_{i,t|0} \cdot MC^n_{i,t|0}$$

代入具体式子

$$\Rightarrow (\varepsilon-1) E_0 \sum_{t=0}^{\infty} (1-\theta)^t \underbrace{\beta^t \left[\left(\frac{Y_0}{Y_t}\right)^{\sigma} \frac{P_0}{P_t} \right]}_{Q_{0,t}} \cdot \underbrace{\left(\frac{P_{i0}^*}{P_t}\right)^{-\varepsilon} Y_t}_{Y_{i,t|0}} = \varepsilon P_{i0}^{*-1} \cdot E_0 \sum_{t=0}^{\infty} (1-\theta)^t \underbrace{\beta^t \left[\left(\frac{Y_0}{Y_t}\right)^{\sigma} \frac{P_0}{P_t} \right]}_{Q_{0,t}} \cdot \underbrace{\left(\frac{P_{i0}^*}{P_t}\right)^{-\varepsilon} Y_t}_{Y_{i,t|0}} \cdot MC^n_{i,t|0}$$

把与t无关的常数项拎出来

$$\Rightarrow (\varepsilon-1) \cancel{Y_0^{\sigma} P_0} \cancel{(P_{i0}^*)^{-\varepsilon}} E_0 \sum_{t=0}^{\infty} [(1-\theta)\beta]^t \cdot P_t^{\varepsilon-1} \cdot Y_t^{1-\sigma} = \varepsilon \cdot P_{i0}^{*-1-\cancel{\varepsilon}} \cancel{Y_0^{\sigma} P_0} E_0 \sum_{t=0}^{\infty} [(1-\theta)\beta]^t \cdot P_t^{\varepsilon-1} \cdot Y_t^{1-\sigma} \cdot MC_{i,t|0}$$

$$\Rightarrow (\varepsilon-1) E_0 \sum_{t=0}^{\infty} [(1-\theta)\beta]^t \cdot P_t^{\varepsilon-1} \cdot Y_t^{1-\sigma} = \varepsilon \cdot (\underline{P_{i0}^*})^{-1} E_0 \sum_{t=0}^{\infty} [(1-\theta)\beta]^t \cdot P_t^{\varepsilon-1} Y_t^{1-\sigma} \cdot MC^n_{i,t|0}$$

最优定价

$$\Rightarrow P_{i0}^* = \frac{\varepsilon}{\varepsilon-1} \frac{E_0 \sum_{t=0}^{\infty} [(1-\theta)\beta]^t \cdot P_t^{\varepsilon-1} Y_t^{1-\sigma} \cdot MC^n_{i,t|0}}{E_0 \sum_{t=0}^{\infty} [(1-\theta)\beta]^t \cdot P_t^{\varepsilon-1} \cdot Y_t^{1-\sigma}}$$

名义边际成本

求得最终的最优定价
包含了垄断竞争和价格粘性的信息

☆ 体现了垄断竞争下的成本加成

☆ 当价格弹性 $\theta = 1$ 时 $1-\theta = 0$ 价格完全弹性.

$0^0 = 1$, $P_{i0}^* = \frac{\varepsilon}{\varepsilon-1} MC^n_{i,t|0}$

可见价格弹性时 价格由MC决定

2、求解菲利普斯曲线

失业率和通胀率的关系

与产出缺口有关

由 $P_{i0}^* = \dfrac{\varepsilon}{\varepsilon-1} \dfrac{E_0 \sum_{t=0}^{\infty} [(1-\theta)\beta]^t \cdot P_t^{\varepsilon-1} Y_t^{1-\sigma} \cdot MC^n_{i,t|0}}{E_0 \sum_{t=0}^{\infty} [(1-\theta)\beta]^t \cdot P_t^{\varepsilon-1} \cdot Y_t^{1-\sigma}}$

书上的形式： $P_t^* = \dfrac{\varepsilon}{\varepsilon-1} \dfrac{E_t \sum_{k=0}^{\infty} [(1-\theta)\beta]^k \cdot P_{t+k}^{\varepsilon-1} Y_{t+k}^{1-\sigma} \cdot MC^n_{i,t+k|t}}{E_t \sum_{k=0}^{\infty} [(1-\theta)\beta]^k \cdot P_{t+k}^{\varepsilon-1} \cdot Y_{t+k}^{1-\sigma}}$

从第t期开始后发展k时期，下标i 即使不写也可以区别是作了价格调整的

$$\underbrace{\frac{1}{P_{t-1}} \cdot P_t^* \cdot E_t \sum_{k=0}^{\infty} [(1-\theta)\beta]^k \cdot P_{t+k}^{\varepsilon-1} \cdot Y_{t+k}^{1-\sigma}}_{LHS} = \underbrace{\frac{\varepsilon}{\varepsilon-1} \cdot \frac{1}{P_{t-1}} E_t \sum_{k=0}^{\infty} [(1-\theta)\beta]^k \cdot P_{t+k}^{\varepsilon-1} Y_{t+k}^{1-\sigma} \cdot MC^n_{i,t+k|t}}_{RHS}$$

左右各4个关于t的变量

进行对数线性化 → $\hat{x}_t = \ln X_t - \ln X = x_t - x$ $x_t = X e^{\hat{x}_t}$

$= \dfrac{X_t - X}{X}$ (一阶泰勒展开)

LHS 的对数线性化：(一阶泰勒展开) $f(x_0) + f'(x_0)(x - x_0)$

$$\sum_{k=0}^{\infty}[(1-\theta)\beta]^k P^{\varepsilon-1}Y^{1-\sigma}+\frac{1}{P}E_t\sum_{k=0}^{\infty}[(1-\theta)\beta]^k P^{\varepsilon-1}Y^{1-\sigma}\cdot(P_t^*-P)-\frac{P}{P^2}E_t\sum_{k=0}^{\infty}[(1-\theta)\beta]^k P^{\varepsilon-1}Y^{1-\sigma}\cdot(P_{t-1}-P)$$

$$+(\varepsilon-1)\cdot\frac{P}{P}\cdot E_t\sum_{k=0}^{\infty}[(1-\theta)\beta]^k P^{\varepsilon-2}Y^{1-\sigma}(P_{t+k}-P)+(1-\sigma)\frac{P}{P}E_t\sum_{k=0}^{\infty}[(1-\theta)\beta]^k P^{\varepsilon-1}Y^{-\sigma}(Y_{t+k}-Y)$$

$$=\sum_{k=0}^{\infty}[(1-\theta)\beta]^k \underline{P^{\varepsilon-1}Y^{1-\sigma}}+E_t\sum_{k=0}^{\infty}[(1-\theta)\beta]^k\underline{P^{\varepsilon-1}Y^{1-\sigma}}\cdot\frac{P_t^*-P}{P}-E_t\sum_{k=0}^{\infty}[(1-\theta)\beta]^k\underline{P^{\varepsilon-1}Y^{1-\sigma}}\frac{P_{t-1}-P}{P}$$

$$+(\varepsilon-1)\cdot E_t\sum_{k=0}^{\infty}[(1-\theta)\beta]^k\underline{P^{\varepsilon-1}Y^{1-\sigma}}\frac{P_{t+k}-P}{P}+(1-\sigma)E_t\sum_{k=0}^{\infty}[(1-\theta)\beta]^k\underline{P^{\varepsilon-1}Y^{1-\sigma}}\cdot\frac{Y_{t+k}-Y}{Y}$$

$$=P^{\varepsilon-1}Y^{1-\sigma}E_t\sum_{k=0}^{\infty}[(1-\theta)\beta]^k\cdot(1+\hat{p}_t^*-\hat{p}_{t-1}+(\varepsilon-1)\hat{p}_{t+k}+(1-\sigma)\hat{y}_{t+k})$$

RHS的对数线性化:

$$\frac{\varepsilon}{\varepsilon-1}\cdot\frac{1}{P}\sum_{k=0}^{\infty}[(1-\theta)\beta]^k\cdot P^{\varepsilon-1}Y^{1-\sigma}\cdot MC^n-\frac{\varepsilon}{\varepsilon-1}\frac{1}{P^2}\sum_{k=0}^{\infty}[(1-\theta)\beta]^k P^{\varepsilon-1}Y^{1-\sigma}MC^n(P_{t-1}-P)$$

$$+\varepsilon\cdot\frac{1}{P}\sum_{k=0}^{\infty}[(1-\theta)\beta]^k\cdot P^{\varepsilon-2}Y^{1-\sigma}MC^n\cdot(P_{t+k}-P)+\frac{\varepsilon(1-\sigma)}{\varepsilon-1}\cdot\frac{1}{P}\sum_{k=0}^{\infty}[(1-\theta)\beta]^k P^{\varepsilon-1}Y^{-\sigma}MC^n(Y_{t+k}-Y)$$

$$+\frac{\varepsilon}{\varepsilon-1}\cdot\frac{1}{P}\sum_{k=0}^{\infty}[(1-\theta)\beta]^k P^{\varepsilon-1}Y^{1-\sigma}(MC^n_{t+k|t}-MC^n)$$

$$=\frac{\varepsilon}{\varepsilon-1}\frac{1}{P}\sum_{k=0}^{\infty}[(1-\theta)\beta]^k\cdot P^{\varepsilon-1}Y^{1-\sigma}MC^n-\frac{\varepsilon}{\varepsilon-1}\cdot\frac{1}{P}\sum_{k=0}^{\infty}[(1-\theta)\beta]^k P^{\varepsilon-1}Y^{1-\sigma}MC^n\hat{p}_t+\varepsilon\frac{1}{P}\sum_{k=0}^{\infty}[(1-\theta)\beta]^k P^{\varepsilon-1}Y^{1-\sigma}MC^n\cdot\hat{p}_{t+k}$$

$$+\frac{\varepsilon(1-\sigma)}{\varepsilon-1}\cdot\frac{1}{P}\sum_{k=0}^{\infty}[(1-\theta)\beta]^k P^{\varepsilon-1}Y^{1-\sigma}MC^n\hat{y}_{t+k}+\frac{\varepsilon}{\varepsilon-1}\cdot\frac{1}{P}\cdot\sum_{k=0}^{\infty}[(1-\theta)\beta]^k P^{\varepsilon-1}Y^{1-\sigma}MC^n\cdot\widehat{mc}^n_{t+k|t}$$

$$=\frac{\varepsilon}{\varepsilon-1}\cdot\frac{1}{P}\cdot P^{\varepsilon-1}\cdot Y^{1-\sigma}MC^n\cdot E_t\sum_{k=0}^{\infty}[(1-\theta)\beta]^k\left[1-\hat{p}_{t-1}+(\varepsilon-1)\hat{p}_{t+k}+(1-\sigma)\hat{y}_{t+k}+\widehat{mc}^n_{t+k|t}\right]$$

LHS = RHS:

$$\cancel{P^{\varepsilon-1}Y^{1-\sigma}}E_t\sum_{k=0}^{\infty}[(1-\theta)\beta]^k\cdot(1+\hat{p}_t^*-\hat{p}_{t-1}+(\varepsilon-1)\hat{p}_{t+k}+(1-\sigma)\hat{y}_{t+k})$$

$$=\cancel{\frac{\varepsilon}{\varepsilon-1}\cdot\frac{1}{P}}\cdot\cancel{P^{\varepsilon-1}\cdot Y^{1-\sigma}}\cancel{MC^n}\cdot E_t\sum_{k=0}^{\infty}[(1-\theta)\beta]^k\left[1-\hat{p}_{t-1}+(\varepsilon-1)\hat{p}_{t+k}+(1-\sigma)\hat{y}_{t+k}+\widehat{mc}^n_{t+k|t}\right]$$

$P=\frac{\varepsilon}{\varepsilon-1}MC^n$ (价格粘性时)

$$\Rightarrow \quad E_t \sum_{k=0}^{\infty} [(1-\theta)\beta]^k \cdot (1 + \hat{p}_t^* - \hat{p}_{t-1} + (\varepsilon-1)\hat{p}_{t+k} + (1-\sigma)\hat{y}_{t+k})$$

$$= E_t \sum_{k=0}^{\infty} [(1-\theta)\beta]^k \cdot (1 - \hat{p}_{t-1} + (\varepsilon-1)\hat{p}_{t+k} + (1-\sigma)\hat{y}_{t+k} + \hat{mc}^n_{t+k|t})$$

因此 $\hat{p}_t^* \, E_t \boxed{\sum_{k=0}^{\infty} [(1-\theta)\beta]^k} = E_t \sum_{k=0}^{\infty} [(1-\theta)\beta]^k \cdot \hat{mc}^n_{t+k|t}$

等比数列求和. $a_1 = 1 \quad q = (1-\theta)\beta$

$$\lim_{n\to\infty} S_n = \lim_{n\to\infty} \frac{a_1(1-q^n)}{1-q} = \lim_{n\to\infty} \frac{1-[(1-\theta)\beta]^n}{1-(1-\theta)\beta} = \frac{1}{1-(1-\theta)\beta}$$

$$\Rightarrow \hat{p}_t^* = [1-(1-\theta)\beta] \; E_t \sum_{k=0}^{\infty} [(1-\theta)\beta]^k \cdot \hat{mc}^n_{t+k|t}$$ 最优定价方程

下面找到通胀率 π 与价格的关系. $\hat{\pi}_t = \theta(\hat{p}_t^* - \hat{p}_{t-1})$

$$\because \; P_t = \left(\int_0^1 P_{it}^{1-\varepsilon} di\right)^{\frac{1}{1-\varepsilon}}$$

$$= \Big[\underbrace{\int_0^\theta P_{it}^{1-\varepsilon} di}_{\text{调价}} + \underbrace{\int_\theta^1 P_{t-1}^{1-\varepsilon} di}_{\text{未调价}}\Big]^{\frac{1}{1-\varepsilon}}$$

$$P_t = \left[\theta (P_t^*)^{1-\varepsilon} + (1-\theta) P_{t-1}^{1-\varepsilon}\right]^{\frac{1}{1-\varepsilon}}$$

$$P_t^{1-\varepsilon} = \theta (P_t^*)^{1-\varepsilon} + (1-\theta) P_{t-1}^{1-\varepsilon} \Rightarrow \frac{P_t^{1-\varepsilon}}{P_{t-1}^{1-\varepsilon}} = \theta\left(\frac{P_t^*}{P_{t-1}}\right)^{1-\varepsilon} + (1-\theta)$$

$$\therefore \underbrace{\pi_t^{1-\varepsilon}}_{LHS} = \underbrace{\theta\left(\frac{P_t^*}{P_{t-1}}\right)^{1-\varepsilon}}_{RHS} + (1-\theta)$$

对数线性化:(对两边一阶泰勒展开)

$$\frac{\partial \frac{P_t^*}{P_{t-1}}}{\partial P_t^*} \qquad \frac{\partial \frac{P_t^*}{P_{t-1}}}{\partial P_{t-1}}$$

$$\pi^{1-\varepsilon} + (1-\varepsilon)\pi^{1-\varepsilon} \cdot \left(\frac{\pi_t - \pi}{\pi}\right) = \theta\left(\frac{P}{P}\right)^{1-\varepsilon} + \theta(1-\varepsilon)\left(\frac{P}{P}\right)^{-\varepsilon} \cdot \left[\frac{1}{P}(P_t^* - P) - \frac{P}{P^2}(P_{t-1} - P)\right] + (1-\theta)$$

$$\Rightarrow \pi^{1-\varepsilon} + (1-\varepsilon)\pi^{1-\varepsilon}\hat{\pi}_t = \cancel{\theta} + \theta(1-\varepsilon)(\hat{p}_t^* - \hat{p}_{t-1}) + (1-\cancel{\theta})$$

注： $\pi_t = \pi = \frac{P_t}{P_t} = 1$ （稳态）

$$\therefore\ \cancel{1} + (1-\cancel{\varepsilon})\hat{\pi}_t = \theta(1-\cancel{\varepsilon})(\hat{p}_t^* - \hat{p}_{t-1}) + \cancel{1}$$

$$\therefore\ \hat{\pi}_t = \theta(\hat{p}_t^* - \hat{p}_{t-1})$$

$$\hookrightarrow\ \hat{p}_t^* - \hat{p}_{t-1} = \frac{1}{\theta}\hat{\pi}_t \quad \Rightarrow \quad \hat{p}_{t+1}^* - \hat{p}_t = \frac{1}{\theta}\hat{\pi}_{t+1}$$

最优定价方程

$$\boxed{\hat{p}_t^* = [1-(1-\theta)\beta]\, E_t \sum_{k=0}^{\infty} [(1-\theta)\beta]^k\, \hat{mc}^{n}_{t+k|t}}$$

$$\hat{p}_t^* - \hat{p}_{t-1} = [1-(1-\theta)\beta]\, E_t \sum_{k=0}^{\infty} [(1-\theta)\beta]^k\, \hat{mc}^{n}_{t+k|t} - \hat{p}_{t-1} = \frac{1}{\theta}\hat{\pi}_t$$

$$= [1-(1-\theta)\beta]\, E_t \sum_{k=0}^{\infty} [(1-\theta)\beta]^k (\hat{mc}^{n}_{t+k|t} - \hat{p}_{t-1})$$

由于 $\sum_{k=0}^{\infty}[(1-\theta)\beta]^k = \frac{1}{1-(1-\theta)\beta}$

$\therefore\ \hat{p}_{t-1} = [1-(1-\theta)\beta]\cdot E_t \sum_{k=0}^{\infty}[(1-\theta)\beta]^k \cdot \hat{p}_{t-1}$

互为倒数.

$$= [1-(1-\theta)\beta]\, E_t \sum_{k=0}^{\infty} [(1-\theta)\beta]^k (\boxed{\hat{mc}^{r}_{t+k|t} + \hat{p}_{t+k}} - \hat{p}_{t-1})$$

$$= \underbrace{[1-(1-\theta)\beta]\, E_t \sum_{k=0}^{\infty} [(1-\theta)\beta]^k\, \hat{mc}^{r}_{t+k|t}}_{=A} + [1-(1-\theta)\beta]\, E_t \sum_{k=0}^{\infty} [(1-\theta)\beta]^k (\hat{p}_{t+k} - \hat{p}_{t-1})$$

列项展开每一项 构造 π_{t+k}

$$= A + [1-(1-\theta)\beta]\, E_t \cdot \Big\{ [\underbrace{(\hat{p}_t - \hat{p}_{t-1})}_{\hat{\pi}_t}] + [(1-\theta)\beta]\cdot(\underbrace{\hat{p}_{t+1} - \hat{p}_t}_{\hat{\pi}_{t+1}} + \underbrace{\hat{p}_t - \hat{p}_{t-1}}_{\hat{\pi}_t}) + [(1-\theta)\beta]^2(\hat{p}_{t+2} - \hat{p}_{t+1} + \hat{p}_{t+1} - \hat{p}_t + \hat{p}_t - \hat{p}_{t-1}) + \cdots \Big\}$$

$$= A + [1-(1-\theta)\beta]\, E_t \cdot \{ \hat{\pi}_t + [(1-\theta)\beta]\cdot(\hat{\pi}_{t+1} + \hat{\pi}_t) + [(1-\theta)\beta]^2(\hat{\pi}_{t+2} + \hat{\pi}_{t+1} + \hat{\pi}_t) + \cdots \}$$

拆开

$$= A + 1\cdot E_t \cdot \{ \hat{\pi}_t + [(1-\theta)\beta]\cdot(\hat{\pi}_{t+1} + \hat{\pi}_t) + [(1-\theta)\beta]^2(\hat{\pi}_{t+2} + \hat{\pi}_{t+1} + \hat{\pi}_t) + \cdots \}$$
$$- (1-\theta)\beta\, E_t \cdot \{ \hat{\pi}_t + [(1-\theta)\beta]\cdot(\hat{\pi}_{t+1} + \hat{\pi}_t) + [(1-\theta)\beta]^2(\hat{\pi}_{t+2} + \hat{\pi}_{t+1} + \hat{\pi}_t) + \cdots \}$$

乘进去.

$$= A + 1\cdot E_t \cdot \{ \hat{\pi}_t + [(1-\theta)\beta]\cdot(\hat{\pi}_{t+1} + \cancel{\hat{\pi}_t}) + [(1-\theta)\beta]^2(\hat{\pi}_{t+2} + \cancel{\hat{\pi}_{t+1} + \hat{\pi}_t}) + \cdots \}$$
$$- \cancel{(1-\theta)\beta\, \hat{\pi}_t} - \cancel{[(1-\theta)\beta]^2(\hat{\pi}_{t+1} + \hat{\pi}_t)} - [(1-\theta)\beta]^3(\cancel{\hat{\pi}_{t+2} + \hat{\pi}_{t+1}} + \hat{\pi}_t) - \cdots \}$$

$$= A + E_t \{ \hat{\pi}_t + (1-\theta)\beta\, \hat{\pi}_{t+1} + [(1-\theta)\beta]^2 \hat{\pi}_{t+2} + \cdots \}$$

$$= A + E_t \sum_{k=0}^{\infty} [(1-\theta)\beta]^k\, \hat{\pi}_{t+k}$$

$$= [1-(1-\theta)\beta] E_t \sum_{k=0}^{\infty} [(1-\theta)\beta]^k \hat{mc}^r_{t+k|t} + E_t \sum_{k=0}^{\infty} [(1-\theta)\beta]^k \hat{\pi}_{t+k}$$

拆成 $k=0$ 和 $k=1 \to \infty$

$$= [1-(1-\theta)\beta]\, \hat{mc}^r_{t|t} + \hat{\pi}_t$$

$k=0$

$$+ [1-(1-\theta)\beta] E_t \sum_{k=1}^{\infty} [(1-\theta)\beta]^k \hat{mc}^r_{t+k|t} + E_t \sum_{k=1}^{\infty} [(1-\theta)\beta]^k \hat{\pi}_{t+k}$$

$k=1 \to \infty$

$k'=0=k-1$

$\therefore k=k'+1$

$k=0 \to \infty$

$$= [1-(1-\theta)\beta]\, \hat{mc}^r_{t|t} + \hat{\pi}_t$$

$$+ [1-(1-\theta)\beta] E_t \sum_{k=0}^{\infty} [(1-\theta)\beta]^{k+1} \hat{mc}^r_{t+k+1|t} + E_t \sum_{k=0}^{\infty} [(1-\theta)\beta]^{k+1} \hat{\pi}_{t+k+1}$$

$$= [1-(1-\theta)\beta]\, \hat{mc}^r_{t|t} + \hat{\pi}_t$$

$$+ (1-\theta)\beta \cdot \left\{ [1-(1-\theta)\beta] E_t \sum_{k=0}^{\infty} [(1-\theta)\beta]^k \hat{mc}^r_{t+k+1|t} + E_t \sum_{k=0}^{\infty} [(1-\theta)\beta]^k \hat{\pi}_{t+k+1} \right\}$$

$$\begin{cases} \hat{p}^*_t - \hat{p}_{t-1} = [1-(1-\theta)\beta] E_t \sum_{k=0}^{\infty} [(1-\theta)\beta]^k \hat{mc}^r_{t+k|t} + E_t \sum_{k=0}^{\infty} [(1-\theta)\beta]^k \hat{\pi}_{t+k} \\ \hat{p}^*_{t+1} - \hat{p}_t = [1-(1-\theta)\beta] E_t \sum_{k=0}^{\infty} [(1-\theta)\beta]^k \hat{mc}^r_{t+k+1|t} + E_t \sum_{k=0}^{\infty} [(1-\theta)\beta]^k \hat{\pi}_{t+k+1} \end{cases}$$

$$\Rightarrow \hat{p}^*_t - \hat{p}_{t-1} = [1-(1-\theta)\beta]\, \hat{mc}^r_{t|t} + \hat{\pi}_t + (1-\theta)\beta \cdot E_t (\hat{p}^*_{t+1} - \hat{p}_t)$$

$\frac{1}{\theta}\hat{\pi}_{t+1}$

$$\hat{\pi}_t = \theta(\hat{p}^*_t - \hat{p}_{t-1}) \Leftrightarrow \hat{p}^*_t - \hat{p}_{t-1} = \frac{1}{\theta}\hat{\pi}_t$$

$$\therefore \frac{1}{\theta}\hat{\pi}_t = [1-(1-\theta)\beta]\, \hat{mc}^r_{t|t} + \hat{\pi}_t + (1-\theta)\beta \cdot E_t \cdot \frac{1}{\theta}\hat{\pi}_{t+1}$$

两边同乘 θ : $\hat{\pi}_t = \theta[1-(1-\theta)\beta]\, \hat{mc}^r_{t|t} + \theta\hat{\pi}_t + (1-\theta)\beta \cdot E_t \hat{\pi}_{t+1}$

$$(1-\theta)\hat{\pi}_t = \theta[1-(1-\theta)\beta]\, \hat{mc}^r_{t|t} + (1-\theta)\beta \cdot E_t \hat{\pi}_{t+1}$$

$$\hat{\pi}_t = \frac{\theta[1-(1-\theta)\beta]}{1-\theta}\, \hat{mc}^r_{t|t} + \beta E_t \cdot \hat{\pi}_{t+1}$$

基于规模报酬不变，所以mc相同.

$$\Longrightarrow \boxed{\hat{\pi}_t = \frac{\theta[1-(1-\theta)\beta]}{1-\theta}\, \hat{mc}^r_t + \beta E_t \cdot \hat{\pi}_{t+1}}$$

通胀率