$$\hat{\pi}_t = \frac{\theta[1-(1-\theta)\beta]}{1-\theta}\,\hat{mc}^r_{t|t} + \beta E_t \hat{\pi}_{t+1}$$

$\hat{mc}^r_{t|t} \Leftrightarrow y_t$

通货膨胀方程 ⇓ NK PC线 & Dynamic IS曲线

① 全&粘 法1
② 全→粘 法2

厂商最小化成本（短中期）

$$\min \frac{W_t}{P_t} N_t$$

$$s.t.\quad A_t N_t^{1-\alpha} = Y_t$$

$$\mathcal{L} = \frac{W_t}{P_t} N_t + \lambda_t (Y_t - A_t N_t^{1-\alpha})$$

$\lambda_t = MC^r_t$

FOC:
$$\frac{\partial \mathcal{L}}{\partial N_t} = 0$$

$$\Rightarrow \frac{W_t}{P_t} = MC^r_t\, (1-\alpha) A_t N_t^{-\alpha}$$

$$\Rightarrow MC^r_t = \frac{W_t/P_t}{MPN_t}$$

劳动边际产出

对数线性化

$$\hat{mc}_t = \hat{w}_t - \hat{p}_t - (a_t - \alpha \hat{n}_t)$$

外生

家庭部门最优化 FOC: 劳动供给方程

$$\frac{W_t}{P_t} = C_t^{\sigma} N_t^{\varphi} = Y_t^{\sigma} N_t^{\varphi}$$

$$= \sigma \hat{y}_t + \varphi \hat{n}_t - (a_t - \alpha \hat{n}_t)$$

$$= \sigma \hat{y}_t - a_t + (\varphi+\alpha)\hat{n}_t$$

$$= \sigma \hat{y}_t - a_t + (\varphi+\alpha)\frac{\hat{y}_t - a_t}{1-\alpha}$$

$Y = A_t N_t^{1-\alpha}$

$$= \frac{\sigma(1-\alpha)+(\varphi+\alpha)}{1-\alpha}\hat{y}_t - \frac{1+\varphi}{1-\alpha} a_t$$

Def $\tilde{y}_t = \hat{y}_t - \hat{y}^f_t$

（产出缺口）

弹性价格时对应的产出

稳态时 对应的产出是 $\hat{y}^f_t$

$$0 \overset{?}{=} \hat{mc}^r_t = \frac{\sigma(1-\alpha)+(\varphi+\alpha)}{1-\alpha}\hat{y}^f_t - \frac{1+\varphi}{1-\alpha} a_t \;?$$

$$\Rightarrow \frac{1+\varphi}{1-\alpha} a_t = \frac{\sigma(1-\alpha)+(\varphi+\alpha)}{1-\alpha}\hat{y}^f_t$$

$(1+\varphi)$ $(1+\varphi)$ 外生

回代

⇓

$$\hat{mc}^r_t = \frac{\sigma(1-\alpha)+(\varphi+\alpha)}{1-\alpha}\tilde{y}_t = (\hat{y}_t - \hat{y}^f_t)$$

回代

⇓

New

$$\underbrace{\hat{\pi}_t = \beta E_t \pi_{t+1} + \chi \tilde{y}_t}_{NKPC}, \quad \chi = \frac{\theta[1-(1-\theta)\beta]}{1-\theta}\cdot\frac{\sigma(1-\alpha)+(\varphi+\alpha)}{1-\alpha}$$

$\pi_{t+1}$

$\tilde{y}_t \uparrow$ : 经济过热 $\Rightarrow \hat{\pi}_t \uparrow$

## Dynamic IS

名义利率

$$\hat{c}_t = E_t \hat{c}_{t+1} - \frac{1}{\sigma}(i_t - E_t \hat{\pi}_{t+1})$$

均衡 $\Rightarrow$ $\hat{y}_t = E_t \hat{y}_{t+1} - \frac{1}{\sigma}(i_t - E_t \hat{\pi}_{t+1})$

Def $\hat{y}_t^f = E_t \hat{y}_{t+1}^f - \frac{1}{\sigma}(i_t^f - 0)$

自然利率

$$\tilde{y}_t = \hat{y}_t - \hat{y}_t^f = E_t \tilde{y}_{t+1} - \frac{1}{\sigma}(i_t - i_t^f - E_t \hat{\pi}_{t+1}) \quad \text{DIS}$$

外生　　外生

DNK model
- NKPC
- DIS

&

Taylor Rule
- 货币供给规则
- 利率规则 ✓

$\hat{y}_t, i_t, \hat{\pi}_{t+1}$ (内生变量)

$$\begin{cases} \hat{\pi}_t = \beta E_t \pi_{t+1} + \kappa \tilde{y}_t \\ \tilde{y}_t = E_t \tilde{y}_{t+1} - \frac{1}{\sigma}(i_t - \cancel{i_t^f} - E_t \hat{\pi}_{t+1}) \\ m_t = \hat{p}_t + \hat{y}_t - \eta \hat{i}_t \end{cases}$$

假设为0

( MIU 货币进入效用函数的 FOC) ← 货币供给规则

外生　$\Delta m_t = \rho_m \Delta m_{t-1} + \varepsilon_t^m$

## 法二. 推导 NKPC

1. 先垄断厂商

相对价格

$$\max_{P_{it}} \pi_{it}^r = (1-\tau_t)\left(\frac{P_{it}}{P_t}\right) Y_{it} - \frac{W_t}{P_t} N_{it}$$

$$\text{s.t.} \quad Y_{it} = A_t N_{it}^{1-\alpha}, \quad \alpha = 0$$

$$Y_{it} = \left(\frac{P_{it}}{P_t}\right)^{-\varepsilon} Y_t$$

$$\pi_{it}^r = (1-\tau_t)\frac{P_{it}}{P_t}\left(\frac{P_{it}}{P_t}\right)^{-\varepsilon} Y_t - \frac{W_t}{P_t} \cdot \left(\frac{P_{it}}{P_t}\right)^{-\varepsilon} \frac{Y_t}{A_t}$$

$$= (1-\tau_t)\left(\frac{P_{it}}{P_t}\right)^{1-\varepsilon} Y_t - \frac{W_t}{P_t} \cdot \left(\frac{P_{it}}{P_t}\right)^{-\varepsilon} \frac{Y_t}{A_t}$$

FOC: $\frac{\partial \pi_{it}^r}{\partial P_{it}} = 0 \Rightarrow \frac{P_{it}^*}{P_t} = \frac{\varepsilon}{\varepsilon-1} \cdot \frac{1}{1-\tau_t} \cdot \frac{W_t/P_t}{A_t}$　$MC_t^r$　劳动供给曲线

$$\Rightarrow \frac{P_{it}^*}{P_t} = \frac{\varepsilon}{\varepsilon-1} \cdot \frac{1}{1-\tau_t} \cdot \frac{Y_t^\sigma N_t^\varphi}{A_t}$$

$$= \underbrace{\frac{\varepsilon}{\varepsilon-1}\cdot\frac{1}{1-\tau_t}\cdot\frac{Y_t^{\sigma}}{A_t}}_{B}\cdot\left(\int_0^1 N_{it}\,di\right)^{\varphi}$$

$$= B\left(\int_0^1 \frac{Y_{it}}{A_t}\,di\right)^{\varphi}$$

$$= \frac{\varepsilon}{\varepsilon-1}\cdot\frac{1}{1-\tau_t}\cdot\frac{Y_t^{\sigma}}{A_t^{1+\varphi}}\left(\int_0^1 Y_{it}\,di\right)^{\varphi}$$

$$= \frac{\varepsilon}{\varepsilon-1}\cdot\frac{1}{1-\tau_t}\cdot\frac{Y_t^{\sigma}}{A_t^{1+\varphi}}\left[\int_0^1 \left(\frac{P_{it}^*}{P_t}\right)^{-\varepsilon} Y_t\,di\right]^{\varphi}$$

$$= \frac{\varepsilon}{\varepsilon-1}\cdot\frac{1}{1-\tau_t}\cdot\frac{Y_t^{\sigma+\varphi}}{A_t^{1+\varphi}}\left[\int_0^1 \left(\frac{P_{it}^*}{P_t}\right)^{-\varepsilon} di\right]^{\varphi}$$

$$= \frac{\varepsilon}{\varepsilon-1}\cdot\frac{1}{1-\tau_t}\cdot\frac{Y_t^{\sigma+\varphi}}{A_t^{1+\varphi}}\left(\frac{P_{it}^*}{P_t}\right)^{-\varepsilon\varphi}$$

价格弹性时 $\frac{P_{it}^*}{P_t} = \frac{P_t^*}{P_t} = 1$

$$1 = \frac{\varepsilon}{\varepsilon-1}\cdot\frac{1}{1-\tau_t}\cdot\frac{Y_t^{\sigma+\varphi}}{A_t^{1+\varphi}} \qquad \Rightarrow Y_t^f = ?$$

$$\left(\frac{P_{it}^*}{P_t}\right)^{1+\varepsilon\varphi} = \frac{\varepsilon}{\varepsilon-1}\cdot\frac{1}{1-\tau_t}\cdot\frac{Y_t^{\sigma+\varphi}}{A_t^{1+\varphi}}$$

小写表 ln

对数线性化 取对数 $(1+\varepsilon\varphi)(p_t^* - p_t) = \ln\frac{\varepsilon}{\varepsilon-1} + \ln\frac{1}{1-\tau_t} + (\sigma+\varphi)y_t - (1+\varphi)a_t$

$$\Rightarrow p_t^* = p_t + \frac{\sigma+\varphi}{1+\varepsilon\varphi}y_t - \frac{1+\varphi}{1+\varepsilon\varphi}a_t + \frac{1}{1+\varepsilon\varphi}\ln\frac{\varepsilon}{\varepsilon-1} - \frac{1}{1+\varepsilon\varphi}\ln(1-\tau_t)$$

减去稳态

$$\hat{p}_t^* = p_t^* - p^*$$

$$= \underbrace{\hat{p}_t + \frac{\sigma+\varphi}{1+\varepsilon\varphi}\hat{y}_t - \frac{1+\varphi}{1+\varepsilon\varphi}a_t}_{C} - \frac{1}{1+\varepsilon\varphi}\left[\ln(1-\tau_t) - \ln(1-\tau)\right]$$

$$= C - \frac{1}{1+\varepsilon\varphi}\left[\cancel{\ln(1-\tau)} - \frac{\tau}{1-\tau}(\tau_t-\tau) - \cancel{\ln(1-\tau)}\right]$$

$$= \hat{p}_t + \frac{\sigma+\varphi}{1+\varepsilon\varphi}\hat{y}_t - \frac{1+\varphi}{1+\varepsilon\varphi}a_t - \frac{\tau}{(1+\varepsilon\varphi)(1-\tau)}\hat{\tau}_t$$

$$0 = \frac{\sigma+\varphi}{1+\varepsilon\varphi}\hat{y}_t^f - \frac{1+\varphi}{1+\varepsilon\varphi}a_t - \frac{\tau}{(1+\varepsilon\varphi)(1-\tau)}\hat{\tau}_t$$

$$\Rightarrow \hat{y}_t^f = \frac{1}{\sigma+\varphi}\left[(1+\varphi)a_t + \frac{\tau}{1-\tau}\hat{\tau}_t\right]$$

$$\Rightarrow \hat{p}_t^* = \hat{p}_t + \gamma(\hat{y}_t - \hat{y}_t^f), \quad \gamma = \frac{\sigma+\varphi}{1+\varepsilon\varphi}$$

desired price

2. 价格粘性 — prob = $\theta$

$$\hat{p}_t^{\Delta} = (1-\theta\beta)\sum_{k=0}^{\infty}(\theta\beta)^k E_t\hat{p}_{t+k}^{*}$$

调整价格

$$= (1-\theta\beta)\hat{p}_t^{*} + (1-\theta\beta)\theta\beta E_t\hat{p}_{t+1}^{*} + (1-\theta\beta)(\theta\beta)^2 E_t\hat{p}_{t+2}^{*} + \cdots$$

$$\hat{p}_t = (1-\theta)\sum_{k=0}^{\infty}\theta^k\hat{p}_{t-k}^{\Delta} = (1-\theta)\hat{p}_t^{\Delta} + \theta\hat{p}_{t-1}$$

$$\begin{cases} \hat{p}_t^{*} = \hat{p}_t + \gamma(\hat{y}_t - \hat{y}_t^f), \quad \gamma = \dfrac{\sigma+\varphi}{1+\varepsilon\varphi} & \text{desired price} \\ \hat{p}_t^{\Delta} = (1-\theta\beta)\sum_{k=0}^{\infty}(\theta\beta)^k E_t\hat{p}_{t+k}^{*} & \text{调整价格方程} \\ \hat{p}_t = (1-\theta)\sum_{k=0}^{\infty}\theta^k\hat{p}_{t-k}^{\Delta} = (1-\theta)\hat{p}_t^{\Delta} + \theta\hat{p}_{t-1} \end{cases}$$

$\boxed{\text{法二 推导 NKPC}}$

$$\hat{p}_t = (1-\theta)\hat{p}_t^{\Delta} + \theta\hat{p}_{t-1}$$

$$(1-\theta)\hat{p}_t^{\Delta} = \hat{p}_t - \theta\hat{p}_{t-1}$$

向前1期 $\quad (1-\theta)E_t\hat{p}_{t+1}^{\Delta} = E_t\hat{p}_{t+1} - \theta\hat{p}_t$

$$\hat{p}_t^{\Delta} = (1-\theta\beta)\hat{p}_t^{*} + \theta\beta E_t\hat{p}_{t+1}^{\Delta}$$

$$\Downarrow$$

$$(1-\theta)\hat{p}_t^{\Delta} = (1-\theta)(1-\theta\beta)\hat{p}_t^{*} + (1-\theta)\theta\beta E_t\hat{p}_{t+1}^{\Delta}$$

$$(1-\theta)\hat{p}_t^{\Delta} = (1-\theta)(1-\theta\beta)\left[\hat{p}_t + \gamma(\hat{y}_t - \hat{y}_t^f)\right] + (1-\theta)\theta\beta E_t\hat{p}_{t+1}^{\Delta}$$

$$\Downarrow \qquad\qquad\qquad \Downarrow$$

$$\hat{p}_t - \theta\hat{p}_{t-1} = (1-\theta)(1-\theta\beta)\left[\hat{p}_t + \gamma(\hat{y}_t - \hat{y}_t^f)\right] + \theta\beta E_t\hat{p}_{t+1} - \theta^2\beta\hat{p}_t$$

合并同类项 $\quad \underbrace{[1+\theta^2\beta-(1-\theta)(1-\theta\beta)]\hat{p}_t}_{=(\theta\beta+\theta)\hat{p}_t} - \theta\hat{p}_{t-1} = (1-\theta)(1-\theta\beta)[\hat{p}_t + \gamma(\hat{y}_t - \hat{y}_t^f)] + \theta\beta E_t\hat{p}_{t+1}$

$$(\theta\beta+\theta)\hat{p}_t - \theta\hat{p}_{t-1} - \theta\beta\hat{p}_t = (1-\theta)(1-\theta\beta)[\gamma(\hat{y}_t - \hat{y}_t^f)] + \theta\beta(E_t\hat{p}_{t+1} - \hat{p}_t)$$

$$\theta(\hat{p}_t - \hat{p}_{t-1}) = (1-\theta)(1-\theta\beta)[\gamma(\hat{y}_t - \hat{y}_t^f)] + \theta\beta(E_t\hat{p}_{t+1} - \hat{p}_t)$$

$$\theta\hat{\pi}_t = (1-\theta)(1-\theta\beta)[\gamma(\hat{y}_t - \hat{y}_t^f)] + \theta\beta E_t\hat{\pi}_{t+1}$$

$$\hat{\pi}_t = \frac{(1-\theta)(1-\theta\beta)}{\theta}[\gamma(\hat{y}_t - \hat{y}_t^f)] + \beta E_t\hat{\pi}_{t+1}$$

$\tilde{x}$ $\Downarrow$ $\hat{\pi}_t = \mathcal{K}\tilde{y} + \beta E_t\hat{\pi}_{t+1}$

2) + 实际货币余额方程 $\Rightarrow$ $m_t = \hat{p}_t + \hat{y}_t$

$$\left\{ \hat{\pi}_t = \mathcal{K}\tilde{y}_t + \beta E_t\hat{\pi}_{t+1} \right.$$

$\Rightarrow \hat{y}_t$

设 $\hat{y}^f_t = 0$

$$\hat{p}_t - \hat{p}_{t-1} = \beta(m_t - \hat{p}_t) + E_t(\hat{p}_{t+1} - \hat{p}_t)$$

$$\Rightarrow E_t\hat{p}_{t+1} - (2+\beta)\hat{p}_t + \hat{p}_{t-1} = -\beta m_t$$

$$\Rightarrow \left[\underbrace{F^2 - (2+\beta)F + 1}\right] L\hat{p}^e_t = -\beta m_t$$

$\lambda_1 + \lambda_2 = 2+\beta$ 设 $\lambda_1 = \theta$, $\lambda_2 = \frac{1}{\theta}$ $\Rightarrow \beta = \frac{(\theta-1)^2}{\theta}$ $F = L^{-1}$

$\lambda_1\lambda_2 = 1$

$$(1-\lambda_1 F)(1-\lambda_2 F)L\hat{p}^e_t = -\beta m_t$$

同乘 $(-\theta)$

$$(1-\theta F)(1-\theta L)\hat{p}^e_t = (-\theta)(-\beta)m_t$$

$$(1-\theta L)\hat{p}^e_t = (1-\theta)^2\underbrace{(1-\theta F)^{-1}}m_t$$

$\parallel$ $(1+\theta F + \theta^2 F^2 + \cdots)$ 等比数列

$$\Rightarrow \hat{p}_t = \theta\hat{p}_{t-1} + (1-\theta)^2\sum_{i=0}^{\infty}\theta^i E_t m_{t+i}$$

利率规则

3) — MP — PC

$$\left\{\begin{array}{l} \hat{\pi}_t = \mathcal{K}\tilde{y}_t + \beta E_t\hat{\pi}_{t+1} \\ \tilde{y}_t = E_t\tilde{y}_{t+1} - \frac{1}{\theta}(\hat{i}_t - E_t\hat{\pi}_{t+1}) \\ \hat{i}_t = \phi_\pi\hat{\pi}_t + \phi_y\tilde{y}_t + v_t \end{array}\right.$$

$\Rightarrow \hat{y}_t$

设 $\hat{y}^f_t = 0$

Taylor Rules $v_t = \rho_v v_{t-1} + \varepsilon^v_t$

B-K 条件

$$\hat{y}_t = E_t\hat{y}_{t+1} - \frac{1}{\theta}\left[(\phi_\pi\hat{\pi}_t + \phi_y\tilde{y}_t + v_t) - E_t\hat{\pi}_{t+1}\right]$$

$$= E_t\hat{y}_{t+1} - \frac{1}{\theta}\left\{[\phi_\pi(\beta E_t\hat{\pi}_{t+1} + \mathcal{K}\tilde{y}_t) + \phi_y\tilde{y}_t + v_t] - E_t\hat{\pi}_{t+1}\right\}$$

$$= E_t\tilde{y}_{t+1} + \frac{1+\beta\phi_\pi}{\sigma}E_t\hat{\pi}_{t+1} - \frac{\phi_\pi\mathcal{K}+\phi_y}{\sigma}\tilde{y}_t - \frac{v_t}{\sigma}$$

$$\hat{y}_t = \frac{1}{\sigma + \phi_\pi \kappa + \phi_y}\left[\sigma E_t \tilde{y}_{t+1} + (1-\beta\phi_\pi) E_t \hat{\pi}_{t+1} - v_t\right]$$

$$\hat{\pi}_t = \beta E_t \hat{\pi}_{t+1} + \kappa \tilde{y}_t$$

$$= \beta E_t \hat{\pi}_{t+1} + \frac{\kappa}{\sigma + \phi_\pi \kappa + \phi_y}\left[\sigma E_t \tilde{y}_{t+1} + (1-\beta\phi_\pi) E_t \hat{\pi}_{t+1} - v_t\right]$$

$$= \frac{1}{\sigma + \phi_\pi \kappa + \phi_y}\left\{\kappa \sigma E_t \tilde{y}_{t+1} + [\kappa + \beta(\sigma + \phi_y)] E_t \hat{\pi}_{t+1} - \kappa v_t\right\}$$

随机前瞻一阶差分系统

$$\Rightarrow \begin{bmatrix} \tilde{y}_t \\ \hat{\pi}_t \end{bmatrix} = \frac{1}{\sigma + \phi_\pi \kappa + \phi_y}\begin{bmatrix} \sigma & 1-\beta\phi_\pi \\ \kappa\sigma & \kappa + \beta(\sigma + \phi_y) \end{bmatrix}\begin{bmatrix} E_t \tilde{y}_{t+1} \\ E_t \hat{\pi}_{t+1} \end{bmatrix} - \frac{1}{\sigma + \phi_\pi \kappa + \phi_y}\begin{bmatrix} 1 \\ \kappa \end{bmatrix} v_t$$

$$\begin{bmatrix} \tilde{y}_t \\ \hat{\pi}_t \end{bmatrix} = A \begin{bmatrix} E_t \tilde{y}_{t+1} \\ E_t \hat{\pi}_{t+1} \end{bmatrix} - B v_t$$

待定系数法：

$$\begin{cases} \tilde{y}_t = \psi_{yv} v_t \\ \hat{\pi}_t = \psi_{\pi v} v_t \end{cases}$$

代入 PC $\quad \hat{\pi}_t = \beta E_t \hat{\pi}_{t+1} + \kappa \tilde{y}_t$

$$\psi_{\pi v} v_t = \beta E_t \psi_{\pi v} v_{t+1} + \kappa \psi_{yv} v_t$$

$$= \beta E_t \psi_{\pi v}(\rho_v v_t + \varepsilon^v_{t+1}) + \kappa \psi_{yv} v_t$$

$$= \beta \psi_{\pi v} \rho_v v_t + \kappa \psi_{yv} v_t$$

$$\Rightarrow (1-\beta\rho_v)\psi_{\pi v} v_t = \kappa \psi_{yv} v_t$$

$$\Rightarrow \psi_{yv} = \frac{1-\beta\rho_v}{\kappa}\psi_{\pi v}$$

代入 DIS $\quad \tilde{y}_t = E_t \tilde{y}_{t+1} - \frac{1}{\sigma}(\hat{i}_t - E_t \hat{\pi}_{t+1})$ 代入 $\hat{i}_t$

$$= E_t \tilde{y}_{t+1} - \frac{1}{\sigma}\left[(\phi_\pi \hat{\pi}_t + \phi_y \tilde{y}_t + v_t) - E_t \hat{\pi}_{t+1}\right]$$

$$\psi_{yv} v_t = \psi_{yv} E_t(\rho_v v_t + \varepsilon^v_{t+1}) - \frac{1}{\sigma}\left[\phi_\pi \psi_{\pi v} v_t + \phi_y \psi_{yv} v_t + v_t - \psi_{\pi v} E_t(\rho_v v_t + \varepsilon^v_{t+1})\right]$$

$$= \frac{\sigma\rho_v - \phi_y}{\sigma}\psi_{yv} v_t - \frac{\phi_\pi - \rho_v}{\sigma}\psi_{\pi v} v_t - \frac{1}{\sigma} v_t$$

$$\sigma \psi_{yv} = (\sigma \rho_v - \phi_y)\psi_{yv} - (\phi_x - \rho_v)\psi_{xv} - 1$$

$$[\sigma(1-\rho_v) + \phi_y]\underbrace{\psi_{yv}}_{= \frac{1-\beta\rho_v}{\alpha}\psi_{xv}} + (\phi_x - \rho_v)\psi_{xv} = -1$$

$$\Downarrow$$

$$\psi_{xv} = \frac{-\alpha}{(1-\beta\rho_v)[\sigma(1-\rho_v)+\phi_y] + \alpha(\phi_x - \rho_v)} = -\alpha\Lambda_v, \ \Lambda_v > 0$$

$$\psi_{yv} = \frac{1-\beta\rho_v}{\alpha}\psi_{xv} = (1-\beta\rho_v)\Lambda_v.$$