西方经济学 2205070031004 姚思怡 [HW5]

新凯恩斯模型

产品市场垄断竞争 → 厂商/产品异质性

→ 关键的行为方程有微观基础

→ 理性预期

→ 货币中性 强调短期

家庭

Step 1: 确定一篮子商品

法1:

$$\max_{C_{it}} \left(\int_0^1 C_{it}^{\frac{\varepsilon-1}{\varepsilon}} di\right)^{\frac{\varepsilon}{\varepsilon-1}}$$

产品 第t期 同期替代弹性

$$\max_{C_{it}} \left(\frac{1}{I}\sum_{i=1}^{I} C_{it}^{\frac{1}{1+\lambda_t}}\right)^{1+\lambda_t} \triangleq C_t$$

$$s.t. \quad \int_0^1 P_{it} C_{it}\, di \le Z_t$$

支出

$$s.t. \quad \sum_{i=1}^{I} P_{it} C_{it} \le Z_t$$

√ 法2:

$$\min \int_0^1 P_{it} C_{it}\, di$$

$$\min \sum_{i=1}^{I} P_{it} C_{it}$$

$$s.t. \quad \left(\int_0^1 C_{it}^{\frac{\varepsilon-1}{\varepsilon}} di\right)^{\frac{\varepsilon}{\varepsilon-1}} \ge C_t$$

$$s.t. \quad \left(\frac{1}{I}\sum_{i=1}^{I} C_{it}^{\frac{1}{1+\lambda_t}}\right)^{1+\lambda_t} \ge C_t$$

$$\mathcal{L} = \int_0^1 P_{it} C_{it}\, di + P_t\left[C_t - \left(\int_0^1 C_{it}^{\frac{\varepsilon-1}{\varepsilon}} di\right)^{\frac{\varepsilon}{\varepsilon-1}}\right]$$

影子价格 (Lagrange 乘子)

$$\Rightarrow P_{it} = P_t \frac{\varepsilon}{\varepsilon-1}\left(\int_0^1 C_{it}^{\frac{\varepsilon-1}{\varepsilon}} di\right)^{\frac{\varepsilon}{\varepsilon-1}-1} \frac{\varepsilon-1}{\varepsilon} C_{it}^{\frac{\varepsilon-1}{\varepsilon}-1}$$

配出 C_t

$$P_{it} = P_t\left[\left(\int_0^1 C_{it}^{\frac{\varepsilon-1}{\varepsilon}} di\right)^{\frac{\varepsilon}{\varepsilon-1}}\right]^{\frac{1}{\varepsilon}} C_{it}^{-\frac{1}{\varepsilon}}$$

$$\frac{P_{it}}{P_t} = \left(\frac{C_{it}}{C_t}\right)^{-\frac{1}{\varepsilon}}$$

$$C_{it} = \left(\frac{P_{it}}{P_t}\right)^{-\varepsilon} C_t$$

$$\int_0^1 P_{it} C_{it}\, di = P_t C_t$$

$$\Rightarrow \int_0^1 P_{it}\left[\left(\frac{P_{it}}{P_t}\right)^{-\varepsilon} C_t\right] di = P_t C_t$$

$$P_t = \left(\int_0^1 P_{it}^{1-\varepsilon} di\right)^{\frac{1}{1-\varepsilon}}$$

Step 2: UMP

新预期 主观贴现因子

$$\max E_0 \sum_{t=0}^{\infty} \beta^t u\left(C_t, N_t, \frac{M_t}{P_t}\right)$$

基期效用 function

$C_{it} \ne C_t$ (垄断竞争)

$N_{it} = N_t$ (完全竞争)

$$s.t. \quad \int_0^1 P_{it} C_{it}\, di + M_t + B_t \le M_{t-1} + (1+r_{t-1}) B_{t-1} + \int_0^1 W_{it} N_{it}\, di + T_t$$

$P_t C_t$ $\quad$ $W_t N_t$ $\quad$ transfer (补贴)

Key Assumption: 购买债券一单位

(卖)销售 $\quad + Q_t B_t \quad\quad + B_{t-1} \quad\quad$ (Def: $Q_t = \frac{1}{1+i_t}$)

1) 劳动供给方程

2) 消费 Euler Equation

3) 货币需求方程

⇒ 对数线性化 * HW *

$$u(\cdot) = \begin{cases} \ln C_t \ \cdots \ ? \\ \frac{C_t^{1-\sigma}-1}{1-\sigma} - \frac{N_t^{1+\varphi}}{1+\varphi} + \frac{\left(\frac{M_t}{P_t}\right)^{1-\theta}}{1-\theta} \end{cases}$$

厂商

(垄断竞争 + 价格粘性) → ε (成本加成是 ε 的函数) → θ (概率 · 价格调整的prob · $1-\theta$ → 粘性prob)

$$\max_{P_{it}\ P_{i0}} E_0 \sum_{t=0}^{\infty} Q_{0,t} (1-\theta)^t \left[P_{i0} Y_{it|0} - TC_{it|0}(Y_{it|0}) \right]$$

$$s.t. \quad Y_{it|0} = \left(\frac{P_{i0}}{P_t}\right)^{-\varepsilon} Y_t \quad \text{代入}$$

[Gali] $\quad \max_{P_{it}} E_t \sum_{k=0}^{\infty} Q_{t,t+k} (1-\theta)^k \left[P_{it} Y_{i,t+k|t} - TC_{i,t+k|t}(Y_{i,t+k|t}) \right]$

$$s.t. \quad Y_{i,t+k|t} = \left(\frac{P_{it}}{P_{t+k}}\right)^{-\varepsilon} Y_{t+k}$$

$$\pi_0^n = E_0 \sum_{t=0}^{\infty} (1-\theta)^t Q_{0,t} \left\{ P_{i0} \left(\frac{P_{i0}}{P}\right)^{-\varepsilon} Y_t - TC_{it}\left[\left(\frac{P_{i0}}{P_t}\right)^{-\varepsilon} Y_t\right] \right\}$$

$$\frac{\partial \pi_0^n}{\partial P_{i0}} = 0 \Rightarrow E_0 \sum_{t=0}^{\infty} (1-\theta)^t Q_{0,t} \left[(1-\varepsilon)\left(\frac{P_{i0}}{P_t}\right)^{-\varepsilon} Y_t - \frac{\partial TC_{it|0}}{\partial Y_{it|0}} \cdot \frac{\partial Y_{it|0}}{\partial P_{i0}} \right] = 0$$

$$E_0 \sum_{t=0}^{\infty} (1-\theta)^t Q_{0,t} \left[(1-\varepsilon)\left(\frac{P_{i0}}{P_t}\right)^{-\varepsilon} Y_t - MC_{it|0} (-\varepsilon)\left(\frac{P_{i0}}{P_t}\right)^{-\varepsilon-1} Y_t \frac{1}{P_t} \right] = 0$$

$$E_0 \sum_{t=0}^{\infty} (1-\theta)^t Q_{0,t} \left[(1-\varepsilon) Y_{it|0} + \frac{\varepsilon}{P_t} MC_{it|0} Y_{it|0} \left(\frac{P_{i0}}{P_t}\right)^{-1} \right] = 0$$

$$E_0 \sum_{t=0}^{\infty} (1-\theta)^t Q_{0,t} Y_{it|0} \left[(1-\varepsilon) + \varepsilon MC_{it|0} P_{i0}^{-1} \right] = 0$$

$$E_0 \sum_{t=0}^{\infty} (1-\theta)^t \boxed{Q_{0,t}} Y_{it|0} (1-\varepsilon) = -E_0 \sum_{t=0}^{\infty} (1-\theta)^t \boxed{Q_{0,t}} Y_{it|0}\, \varepsilon\, MC_{it|0} P_{i0}^{-1}$$

$Y_{it|0} = \left(\frac{P_{i0}}{P_t}\right)^{-\varepsilon} Y_t$

$\boxed{Q_{0,t} \Leftarrow \text{Euler Equation}}$ ⇒

$$(\varepsilon-1) E_0 \sum_{t=0}^{\infty} (1-\theta)^t \beta^t \left[\left(\frac{Y_0}{Y_t}\right)^{\sigma} \frac{P_0}{P_t}\right] \left(\frac{P_{i0}}{P_t}\right)^{-\varepsilon} Y_t$$

$$= \varepsilon P_{i0}^{-1} E_0 \sum_{t=0}^{\infty} (1-\theta)^t \beta^t \left[\left(\frac{Y_0}{Y_t}\right)^{\sigma} \frac{P_0}{P_t}\right] \left(\frac{P_{i0}}{P_t}\right)^{-\varepsilon} Y_t\, MC_{it|0}$$

$$\Rightarrow (\varepsilon-1) E_0 \sum_{t=0}^{\infty}[(1-\theta)\beta]^t P_t^{\varepsilon-1} Y_t^{1-\sigma} = \varepsilon P_{i0}^{-1} E_0 \sum_{t=0}^{\infty}[(1-\theta)\beta]^t P_t^{\varepsilon-1} Y_t^{1-\sigma} MC_{it|0}$$

$$\Rightarrow (\varepsilon-1)\cancel{(P_{i0}^*)^{-\varepsilon} P_0 Y_0^{\sigma}}\, E_0 \sum_{t=0}^{\infty}[(1-\theta)\beta]^t P_t^{\varepsilon-1} Y_t^{1-\sigma}$$

$$= \varepsilon (P_{i0}^*)^{-1} \cancel{(P_{i0}^*)^{-\varepsilon}}\, \cancel{P_0 Y_0^{\sigma}}\, E_0 \sum_{t=0}^{\infty}[(1-\theta)\beta]^t P_t^{\varepsilon-1} Y_t^{1-\sigma} MC_{i,t|0}$$

$$\Rightarrow P_{i0}^* = \frac{\varepsilon}{\varepsilon-1} \boxed{\frac{E_0 \sum_{t=0}^{\infty}[(1-\theta)\beta]^t P_t^{\varepsilon-1} Y_t^{1-\sigma} MC_{it|0}^n}{E_0 \sum_{t=0}^{\infty}[(1-\theta)\beta]^t P_t^{\varepsilon-1} Y_t^{1-\sigma}}}$$

定价公式

成本加成

MC

计价格弹性，$\theta=1$，$t=0$

价格粘性 { Calvo 1983, JME; Taylor, 1978, JPE

$\hat{x}_t = \ln X_t - \ln X$

$\hat{x}_t = \frac{X_t - X}{X}$ (一阶 Taylor 展开)

$X_t = X e^{\hat{x}_t}$

对数线性化 ⇒ FC line

$$P_t^* = \frac{\varepsilon}{\varepsilon-1} \cdot \frac{E_t \sum_{k=0}^{\infty}[(1-\theta)\beta]^k P_{t+k}^{\varepsilon-1} Y_{t+k}^{1-\sigma} MC_{i,t+k|t}^n}{E_t \sum_{k=0}^{\infty}[(1-\theta)\beta]^k P_{t+k}^{\varepsilon-1} Y_{t+k}^{1-\sigma}}$$

$$\text{(LHS)}\quad \frac{P_t^*}{P_{t-1}} E_t \sum_{k=0}^{\infty}[(1-\theta)\beta]^k P_{t+k}^{\varepsilon-1} Y_{t+k}^{1-\sigma} = \frac{\varepsilon}{\varepsilon-1} \frac{1}{P_{t-1}} E_t \sum_{k=0}^{\infty}[(1-\theta)\beta]^k P_{t+k}^{\varepsilon-1} Y_{t+k}^{1-\sigma} MC_{t+k|t}^n \quad \text{(RHS)}$$

稳态时 $P_t^* = P_{t-1}$

$$\sum_{k=0}^{\infty}[(1-\theta)\beta]^k P^{\varepsilon-1} Y^{1-\sigma} + \frac{1}{P} E_t \sum_{k=0}^{\infty}[(1-\theta)\beta]^k P^{\varepsilon-1} Y^{1-\sigma} (P_t^* - P) - \frac{P}{P^2} E_t \sum_{k=0}^{\infty}[(1-\theta)\beta]^k P^{\varepsilon-1} Y^{1-\sigma} (P_{t-1} - P)$$

$$+ (\varepsilon-1) E_t \sum_{k=0}^{\infty}[(1-\theta)\beta]^k P^{\varepsilon-2} Y^{1-\sigma} (P_{t+k} - P) + (1-\sigma) E_t \sum_{k=0}^{\infty}[(1-\theta)\beta]^k P^{\varepsilon-1} Y^{-\sigma} (Y_{t+k} - Y)$$

$$LHS = \cancel{P^{\varepsilon-1} Y^{1-\sigma}} E_t \sum_{k=0}^{\infty}[(1-\theta)\beta]^k \left[\cancel{1} + \hat{p}_t^* - \cancel{\hat{p}_{t-1}} + \cancel{(\varepsilon-1)\hat{p}_{t+k}} + \cancel{(1-\sigma)\hat{y}_{t+k}}\right]$$

$$\|$$

$$RHS = \frac{\varepsilon}{\varepsilon-1} \frac{MC^n}{P} \cancel{P^{\varepsilon-1} Y^{1-\sigma}} E_t \sum_{k=0}^{\infty}[(1-\theta)\beta]^k \left[\cancel{1} - \cancel{\hat{p}_{t-1}} + \cancel{(\varepsilon-1)\hat{p}_{t+k}} + \cancel{(1-\sigma)\hat{y}_{t+k}} + \hat{mc}_{t+k|t}\right]$$

$P = \frac{\varepsilon}{\varepsilon-1} MC^n$ (稳态时)　　$\frac{MC^n}{P} = \frac{\varepsilon-1}{\varepsilon}$

$$\Rightarrow \hat{p}_t^* \underbrace{\sum_{k=0}^{\infty}[(1-\theta)\beta]^k}_{= \frac{1}{1-(1-\theta)\beta}} = E_t \sum_{k=0}^{\infty}[(1-\theta)\beta]^k \hat{mc}_{t+k|t}$$

(等比数列求和)

$$\Rightarrow \hat{p}_t^* = [1-(1-\theta)\beta] E_t \sum_{k=0}^{\infty}[(1-\theta)\beta]^k \hat{mc}_{t+k|t}$$

$$P_t = \left(\int_0^1 P_{it}^{1-\varepsilon} di\right)^{\frac{1}{1-\varepsilon}} = \left[\int_0^{\theta} (P_t^*)^{1-\varepsilon} di + \int_{\theta}^1 P_{t-1}^{1-\varepsilon} di\right]^{\frac{1}{1-\varepsilon}}$$

θ调价　不调价 (1-θ)

$$\Rightarrow P_t = \left(\theta P_t^{*\,1-\varepsilon} + (1-\theta) P_{t-1}^{1-\varepsilon}\right)^{\frac{1}{1-\varepsilon}}$$

$$\Rightarrow P_t^{1-\varepsilon} = \theta P_t^{*\,1-\varepsilon} + (1-\theta) P_{t-1}^{1-\varepsilon}$$

总通胀 $= \frac{P_t}{P_{t-1}}$

$$\left(\frac{P_t}{P_{t-1}}\right)^{1-\varepsilon} = \theta \left(\frac{P_t^*}{P_{t-1}}\right)^{1-\varepsilon} + (1-\theta)$$

$$\Rightarrow \pi_t^{1-\varepsilon} = \theta\left(\frac{P_t^*}{P_{t-1}}\right)^{1-\varepsilon} + (1-\theta)$$ ← 通货膨胀与 P_t^* 的关系

对数线性化:

$$\cancel{\pi^{1-\varepsilon}} + (1-\varepsilon)\pi^{1-\varepsilon}\underbrace{(\pi_t - \pi)}_{\pi} = \cancel{\theta} + \theta(1-\varepsilon)\left(\frac{P}{P}\right)^{-\varepsilon}\left[\frac{1}{P}(P_t^* - P) - \frac{P}{P^2}(P_{t-1} - P)\right] + \cancel{(1-\theta)}$$

$1 = \frac{P}{P}$

$$\Rightarrow \hat{\pi}_t = \theta(\hat{p}_t^* - \hat{p}_{t-1})$$

名义→实际

$$\frac{\hat{\pi}_t}{\theta} = \hat{p}_t^* - \hat{p}_{t-1} = [1-(1-\theta)\beta]\, E_t \sum_{k=0}^{\infty}[(1-\theta)\beta]^k \left(mc^n_{t+k/t} - \hat{p}_{t-1}\right)$$

$mc^r_{t+k|t} + \hat{P}_{t+k}$

$$= \underbrace{[1-(1-\theta)\beta]\, E_t \sum_{k=0}^{\infty}[(1-\theta)\beta]^k\, mc^r_{t+k|t}}_{A} + [1-(1-\theta)\beta]\, E_t \sum_{k=0}^{\infty}[(1-\theta)\beta]^k (\hat{p}_{t+k} - \hat{p}_{t-1})$$

△打开

$$= A + [1-(1-\theta)\beta]\, E_t(\underbrace{\hat{p}_t - \hat{p}_{t-1}}_{\hat{\pi}_t}) + [(1-\theta)\beta]\, E_t(\hat{p}_{t+1} - \hat{p}_{t-1}) + [(1-\theta)\beta]^2 E_t(\hat{p}_{t+2} - \hat{p}_{t-1}) + \cdots$$

$-\hat{P}_t + \hat{P}_t$　　$-\hat{P}_{t+1} + \hat{P}_{t+1} - \hat{P}_t + \hat{P}_t$

$$= A + [1-(1-\theta)\beta] E_t\left\{\hat{\pi}_t + [(1-\theta)\beta](\hat{\pi}_{t+1} + \hat{\pi}_t) + [(1-\theta)\beta]^2(\hat{\pi}_{t+2} + \hat{\pi}_{t+1} + \hat{\pi}_t) + \cdots\right\}$$

$$= A + 1 \times E_t\{\cdot\cdot\} - (1-\theta)\beta\, E_t\{\cdot\cdot\}$$

$$= A + E_t\{\cdot\cdot\} - (1-\theta)\beta\hat{\pi}_t - [(1-\theta)\beta]^2 E_t(\hat{\pi}_{t+1} + \hat{\pi}_t) - [(1-\theta)\beta]^3(\hat{\pi}_{t+2} + \hat{\pi}_{t+1} + \hat{\pi}_t) - \cdots$$

$$= A + E_t \sum_{k=0}^{\infty}[(1-\theta)\beta]^k \hat{\pi}_{t+k}$$

$$= [1-(1-\theta)\beta]\, E_t \sum_{k=0}^{\infty}[(1-\theta)\beta]^k\, mc^r_{t+k|t} + E_t \sum_{k=0}^{\infty}[(1-\theta)\beta]^k \hat{\pi}_{t+k}$$

△展开

$$= \underbrace{[1-(1-\theta)\beta]\, mc^r_{t|t} + \hat{\pi}_t}_{k=0,\ \triangleq B} + \boxed{\sum_{k=1}^{\infty}}$$

$$= B + [1-(1-\theta)\beta]\, E_t \sum_{k=0}^{\infty}[(1-\theta)\beta]^{k+1} mc^r_{t+k+1/t} + E_t \sum_{k=0}^{\infty}[(1-\theta)\beta]^{k+1}\hat{\pi}_{t+k+1}$$

$$= B + [(1-\theta)\beta]\left\{[1-(1-\theta)\beta]\, E_t \sum_{k=0}^{\infty}[(1-\theta)\beta]^k mc^r_{t+k+1/t} + E_t \sum_{k=0}^{\infty}[(1-\theta)\beta]^k \hat{\pi}_{t+k+1}\right\}$$

$$\frac{\hat{\pi}_t}{\theta} = [1-(1-\theta)\beta]\, mc^r_{t|t} + \hat{\pi}_t + [1-(1-\theta)\beta]\, \underbrace{E_t(\hat{p}^*_{t+1} - \hat{P}_t)}_{\frac{1}{\theta}E_t\hat{\pi}_{t+1}}$$