The Overlapping Generations Model of Money

Econ 208

Lecture 13

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Introduction

- Need something with better micro foundation than money in the utility function
- The role of confidence in supporting the value of money shd be made explicit
- if people believe fiat money will be worthless then it will be
- Basic assumptions:
  - Economy has an infinite horizon
  - Each person lives for 2 periods
  - Just one good, non-storeable
  - Each young person endowed with $y > 0$
  - Each old person unendowed
  - Utility of generation t: $u_t = u(c_{1t}, c_{2t})$
  - Population growth rate n: $N_t = (1+n)^t$

Equilibrium with no money

- No gains from borrowing or lending
- Therefore no one consumes when old
- Everyone attains

$$u^{autarky} = u\left(y, 0\right)$$

- This is probably socially inefficient because old people are starving and would pay a lot to consume if they could
- Define the "autarky rate of interest" r^A as the slope of an indifference curve in autarky (-1):

$$1 + r^A = \frac{u_1\left(y, 0\right)}{u_2\left(y, 0\right)}$$

  (show diagram)
- Then the autarky equilibrium without money is socially inefficient if $r^A < n$.

Socially Efficient Allocations

- Each period, the social resource constraint is:

$$N_t c_{1t} + N_{t-1} c_{2,t-1} = N_t y$$

Divide by N_t:

$$c_{1t} + \frac{1}{1+n} c_{2,t-1} = y$$

(show diagram)

- This looks like the individual's lifetime budget constraint when the rate of interest is n.
- This means that n is the "biological rate of interest"

Social Inefficiency without Money

- If $n > r^A$ then people are getting less than the biological rate of interest in autarky, so autarky is inefficient
- To make a Pareto improvement from autarky, get each young individual to give up ε and give each old person $(1 + n)\,\varepsilon$. This is feasible and will raise each generation's lifetime utility (see diagram)
- Why does the first welfare theorem fail here? Because not everyone participates
- It depends on having an infinite horizon (Gamov's hotel).
- Social improvement could be implemented by social security scheme.
- The optimal allocation for all but generation 0 is where $r = n$ (show diagram)

Perfect foresight equilibrium with money

Suppose there are M units of some storeable object (token money, fiat money)
Suppose now that each old person at time 1 starts the period holding M/N_0 of money.
Keep M constant over time
Now people can trade the good for money
A perfect foresight equilibrium can have two forms.

1. Autarky is still an equilibrium. If everyone thinks that the tokens will be worthless then they will be, and people will consume their endowments when young.
2. Monetary equilibrium - a time path $\{P_1, P_2, ...\}$.of *finite* prices such that if everyone knows this path will be followed then supply of goods = demand for goods each period.

Monetary Equilibrium

- Each young person chooses (c_{1t}, c_{2t}) to

$$\max u(c_{1t}, c_{2t}) \qquad \text{subj to } P_{t+1}c_{2t} = (y - c_{1t})P_t, \text{ that is:}$$
$$\max u(c_{1t}, c_{2t}) \qquad \text{subj to } c_{1t} + (1+\pi_t)c_{2t} = y, \text{ where}$$

$$1 + \pi_t \equiv P_{t+1}/P_t$$

- In effect, they are saving at the rate of interest r_t defined by

$$1 + r_t = \frac{1}{1+\pi_t}$$

- So the demand for goods by the young is

$$c_{1t} = \widetilde{c}\left(\frac{1}{1+\pi_t}\right)$$

- Each old person at t just spends M/N_{t-1}

Monetary Equilibrium (cont'd)

- The equilibrium condition each period is the social resource constraint:

$$
\begin{aligned}
c_{1t} + \frac{1}{1+n} c_{2,t-1} &= y \\
\widetilde{c}\left(P_t / P_{t+1}\right) + \frac{1}{1+n} \frac{M}{N_{t-1} P_t} &= y \\
\widetilde{c}\left(P_t / P_{t+1}\right) + \frac{M}{N_t P_t} &= y
\end{aligned}
$$

- As usual in foreward looking models this determines P_t in terms of P_{t+1}
- A **stationary** monetary equilibrium is a monetary equilibrium with

$$
\left(c_{1t}, c_{2t}\right) = \left(c_1, c_2\right), \text{ constant}
$$

Stationary Monetary Equilibrium (cont'd)

- In a stationary monetary equilibrium

$$1 + \pi_t = \frac{1}{1+n} \text{ for all t}$$

- This is because the budget constraint requires

$$c_1 + (1 + \pi_t)\, c_2 = y$$

while the equilibrium condition requires

$$c_1 + \frac{1}{1+n} c_2 = y$$

and for this to be monetary we need $c_2 > 0$.

- So a stationary monetary equilibrium delivers the "optimal" allocation, because it allows young people to save at the biological rate of interest
- The quantity theory of money holds (and monetary neutrality) because

$$c_2 = \frac{M}{P_t N_{t-1}} = \text{constant}$$

Existence of stationary monetary equilibrium

- A stationary monetary equilibrium exists if and only if autarky is inefficient
- This is because

$$\text{autarky inefficient } \Leftrightarrow r^A > n \Leftrightarrow \widetilde{c}\,(1+n) < y \Leftrightarrow c_2 > 0$$

  (show diagram)
- So money has a socially useful role to play here
- In effect it solves the problem of absence of double coincidence of wants

Stationary monetary equilibrium with a time-varying money supply

- Suppose each person of generation t is given a transfer equal to:

$$\frac{TR_{t+1}}{N_t}$$

  just before trading at time $t+1$.
- The money supply starts at M_1 and then:

$$M_{t+1} = M_t + TR_{t+1}$$

- Assume constant money growth:

$$TR_{t+1} = \mu M_t, \text{ so } M_{t+1} = (1+\mu) M_t$$

Stationary monetary equilibrium with a time-varying money supply (cont'd)

- Each young person chooses (c_{1t}, c_{2t}) to

$$\max u\left(c_{1t}, c_{2t}\right) \quad \text{subj to } P_{t+1} c_{2t} = \left(y - c_{1t}\right) P_t + TR_{t+1}, \text{ that is:}$$
$$\max u\left(c_{1t}, c_{2t}\right) \quad \text{subj to } c_{1t} + \left(1+\pi_t\right) c_{2t} = y + \left(1+\pi_t\right) tr_{t+1}, \text{ where}$$

$$tr_{t+1} \equiv TR_{t+1} / P_{t+1}$$

- Equilibrium condition:

$$c_2 = \frac{M_t}{N_{t-1} P_t}, \text{ constant } \Leftrightarrow 1 + \pi_t = \frac{1+\mu}{1+n}$$

- First-order condition:

$$\frac{u_1\left(c_1, c_2\right)}{u_2\left(c_1, c_2\right)} = \frac{1}{1+\pi_t} = \frac{1+n}{1+\mu}$$

- An SME will not be efficient if $\mu > 0$, because again people will be receiving less than the biological rate of interest (show diagram)

The Diamond model with money

- Problem with the basic OLG model - no store of value except money
- But this can sometimes be fixed up by adding productive capital as in the Diamond model.
- Assumptions
  - Old people hold capital and money
  - Young endowed with labor
  - firms produce according to $Y_t = F(K_t, N_t)$ where now N_t is the size of gen t.
  - r_t is the real rate of interest on loans to firms
  - w_t the real wage rate

maximize $u(c_1, c_2)$ subject to:

$$\widetilde{k}_{t+1} + (1+\pi_t)\,\widetilde{m}_{t+1} = w_t - c_1 \text{ and}$$
$$c_2 = (1+r_{t+1})\,\widetilde{k}_{t+1} + \widetilde{m}_{t+1}, \text{ where}$$

$\widetilde{m}_{t+1}, \widetilde{k}_{t+1}$ denote per member of gen t

- First-order conditions:

$$\begin{aligned} u_1(c_1, c_2) &= \lambda_1 \\ u_2(c_1, c_2) &= \lambda_2 \\ \lambda_1 &= (1+r_{t+1})\,\lambda_2 \\ (1+\pi_t)\,\lambda_1 &\geq \lambda_2, \; (\widetilde{m}_{t+1} = 0 \text{ if } >) \end{aligned}$$

- So for $\widetilde{m}_{t+1} > 0$ we need

$$(1+\pi_t)(1+r_{t+1}) = 1$$

- In a monetary equilibrium we need the same rate of return on saving in the form of money or capital, so

$$y - c_{1t} = \widetilde{s}\left(w_t, r_{t+1}\right)$$

- A monetary equilibrium, given k_0, is a sequence $\{m_t, k_t, \pi_t\}_0^\infty$ (per gen t) such that for each t, $m_t > 0$ and

$$\begin{aligned}(1+n)\left(k_{t+1} + (1+\pi_t)\, m_{t+1}\right) &= s\left(w\left(k_t\right), f'\left(k_{t+1}\right) - \delta\right) \\ (1+\pi_t)\left(1 + f'\left(k_{t+1}\right) - \delta\right) &= 1 \text{ and} \\ (1+n)\left(1+\pi_t\right) m_{t+1} &= m_t \text{ (constant money supply)}\end{aligned}$$

- A stationary monetary equilibrium is (m, k, π) such that $m > 0$ and

$$\begin{aligned}(1+n)\left(k + (1+\pi)\, m\right) &= s\left(w\left(k\right), f'\left(k\right) - \delta\right) \\ (1+\pi)\left(1 + f'\left(k\right) - \delta\right) &= 1 \text{ and} \\ (1+n)\left(1+\pi\right) &= 1\end{aligned}$$

All necessary condition for SME is

$$f'(k) - \delta = n$$

That is, an SME must achieve the golden-rule capital stock per worker k^g!
Also necessary is

$$s(w(k), n) > (1 + n)k$$

(otherwise $m = 0$).
So a SME exists if and only if **capital overaccumulation** occurs without money.(see diagram)