

# *Chapter 1*

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## *Introduction*

Science is facts; just as houses are made of stones,  
so is science made of facts; but a pile of stones is not  
a house and a collection of facts is not necessarily  
science.

—Henri Poincaré

### **1.1 Background**

The seminal contribution of Kydland and Prescott (1982) marked the crest of a sea change in the way macroeconomists conduct empirical research. Under the empirical paradigm that remained predominant at the time, the focus was either on purely statistical (or reduced-form) characterizations of macroeconomic behavior, or on systems-of-equations models that ignored both general-equilibrium considerations and forward-looking behavior on the part of purposeful decision makers. But the powerful criticism of this approach set forth by Lucas (1976), and the methodological contributions of, for example, Sims (1972) and Hansen and Sargent (1980), sparked a transition to a new empirical paradigm. In this transitional stage, the formal imposition of theoretical discipline on reduced-form characterizations became established. The source of this discipline was a class of models that have come to be known as dynamic stochastic general equilibrium (DSGE) models. The imposition of discipline most typically took the form of “cross-equation restrictions,” under which the stochastic behavior of a set of exogenous variables, coupled with forward-looking behavior on the part of economic decision makers, yield implications for the endogenous stochastic behavior of variables determined by the decision makers. Nevertheless, the imposition of such restrictions was indirect, and reduced-form specifications continued to serve as the focal point of empirical research.

Kydland and Prescott turned this emphasis on its head. As a legacy of their work, DSGE models no longer serve as indirect sources of theoretical discipline to be imposed upon statistical specifications. Instead, they serve directly as the foundation upon which empirical work may be conducted. The methodologies used to implement DSGE models as foundational empirical models have evolved over time and vary considerably.

The same is true of the statistical formality with which this work is conducted. But despite the characteristic heterogeneity of methods used in pursuing contemporary empirical macroeconomic research, the influence of Kydland and Prescott remains evident today.

This book details the use of DSGE models as foundations upon which empirical work may be conducted. It is intended primarily as an instructional guide for graduate students and practitioners, and so contains a distinct how-to perspective throughout. The methodologies it presents are organized roughly following the chronological evolution of the empirical literature in macroeconomics that has emerged following the work of Kydland and Prescott; thus it also serves as a reference guide. Throughout, the methodologies are demonstrated using applications to three benchmark models: a real-business-cycle model (fashioned after King, Plosser, and Rebelo, 1988); a monetary model featuring monopolistically competitive firms (fashioned after Ireland, 2004a); and an asset-pricing model (fashioned after Lucas, 1978).

The empirical tools outlined in the text share a common foundation: a system of nonlinear expectational difference equations derived as the solution of a DSGE model. The strategies outlined for implementing these models empirically typically involve the derivation of approximations of the systems, and then the establishment of various empirical implications of the systems. The primary focus of this book is on the latter component of these strategies: This text covers a wide range of alternative methodologies that have been used in pursuit of a wide range of empirical applications. Demonstrated applications include: parameter estimation, assessments of fit and model comparison, forecasting, policy analysis, and measurement of unobservable facets of aggregate economic activity (e.g., measurement of productivity shocks).

## **1.2 Overview**

This book is divided into three parts. Part I presents foundational material included to help keep the book self-contained. Following this introduction, chapter 2 outlines two preliminary steps often used in converting a given DSGE model into an empirically implementable system of equations. The first step involves the linear approximation of the model; the second step involves the solution of the resulting linearized system. The solution takes the form of a state-space representation for the observable variables featured in the model.

Chapter 3 presents two important preliminary steps often needed for priming data for empirical analysis: removing trends and isolating cycles. The purpose of these steps is to align what is being measured in the data with what is being modelled by the theory. For example, the separation of

trend from cycle is necessary in confronting trending data with models of business cycle activity.

Chapter 4 presents tools used to summarize properties of the data. First, two important reduced-form models are introduced: autoregressive-moving average models for individual time series, and vector autoregressive models for sets of time series. These models provide flexible characterizations of the data that can be used as a means of calculating a wide range of important summary statistics. Next, a collection of popular summary statistics (along with algorithms available for calculating them) are introduced. These statistics often serve as targets for estimating the parameters of structural models, and as benchmarks for judging their empirical performance. Empirical analyses involving collections of summary statistics are broadly categorized as limited-information analyses. Finally, the Kalman filter is presented as a means for pursuing likelihood-based, or full-information, analyses of state-space representations. Part I concludes in chapter 5 with an introduction of the benchmark models that serve as examples in part II.

Part II, composed of chapters 6 through 9, presents the following empirical methodologies: calibration, limited-information estimation, maximum likelihood estimation, and Bayesian estimation. Each chapter contains a general presentation of the methodology, and then presents applications of the methodology to the benchmark models in pursuit of alternative empirical objectives.

Chapter 6 presents the most basic empirical methodology covered in the text: the calibration exercise, as pioneered by Kydland and Prescott (1982). Original applications of this exercise sought to determine whether models designed and parameterized to provide an empirically relevant account of long-term growth were also capable of accounting for the nature of short-term fluctuations that characterize business-cycle fluctuations, summarized using collections of sample statistics measured in the data. More generally, implementation begins with the identification of a set of empirical measurements that serve as constraints on the parameterization of the model under investigation: parameters are chosen to insure that the model can successfully account for these measurements. (It is often the case that certain parameters must also satisfy additional a priori considerations.) Next, implications of the duly parameterized model for an additional set of statistical measurements are compared with their empirical counterparts to judge whether the model is capable of providing a successful account of these additional features of the data. A challenge associated with this methodology arises in judging success, because this second-stage comparison is made in the absence of a formal statistical foundation.

The limited-information estimation methodologies presented in chapter 7 serve as one way to address problems arising from the statistical informality associated with calibration exercises. Motivation for their im-

plementation stems from the fact that there is statistical uncertainty associated with the set of empirical measurements that serve as constraints in the parameterization stage of a calibration exercise. For example, a sample mean has an associated sample standard error. Thus there is also statistical uncertainty associated with model parameterizations derived from mappings onto empirical measurements (referred to generally as statistical moments). Limited-information estimation methodologies account for this uncertainty formally: the parameterizations they yield are interpretable as estimates, featuring classical statistical characteristics. Moreover, if the number of empirical targets used in obtaining parameter estimates exceeds the number of parameters being estimated (i.e., if the model in question is over-identified), the estimation stage also yields objective goodness-of-fit measures that can be used to judge the model's empirical performance. Prominent examples of limited-information methodologies include the generalized and simulated methods of moments (GMM and SMM), and indirect-inference methods.

Limited-information estimation procedures share a common trait: they are based on a subset of information available in the data (the targeted measurements selected in the estimation stage). An attractive feature of these methodologies is that they may be implemented in the absence of explicit assumptions regarding the underlying distributions that govern the stochastic behavior of the variables featured in the model. A drawback is that decisions regarding the moments chosen in the estimation stage are often arbitrary, and results (e.g., regarding fit) can be sensitive to particular choices. Chapters 8 and 9 present full-information counterparts to these methodologies: likelihood-based analyses. Given a distributional assumption regarding sources of stochastic behavior in a given model, chapter 8 details how the full range of empirical implications of the model may be assessed via maximum-likelihood analysis, facilitated by use of the Kalman filter. Parameter estimates and model evaluation are facilitated in a straightforward way using maximum-likelihood techniques. Moreover, given model estimates, the implied behavior of unobservable variables present in the model (e.g., productivity shocks) may be inferred as a by-product of the estimation stage.

A distinct advantage in working directly with structural models is that, unlike their reduced-form counterparts, one often has clear a priori guidance concerning their parameterization. For example, specifications of subjective annual discount rates that exceed 10% may be dismissed out-of-hand as implausible. This motivates the subject of chapter 9, which details the adoption of a Bayesian perspective in bringing full-information procedures to bear in working with structural models. From the Bayesian perspective, a priori views on model parameterization may be incorporated formally in the empirical analysis, in the form of a prior distribution. Cou-

pled with the associated likelihood function via Bayes' Rule, the corresponding posterior distribution may be derived; this conveys information regarding the relative likelihood of alternative parameterizations of the model, conditional on the specified prior and observed data. In turn, conditional statements regarding the empirical performance of the model relative to competing alternatives, the implied behavior of unobservable variables present in the model, and likely future trajectories of model variables may also be derived. A drawback associated with the adoption of a Bayesian perspective in this class of models is that posterior analysis must be accomplished via the use of sophisticated numerical techniques; special attention is devoted to this problem in the chapter.

Part III outlines how nonlinear model approximations can be used in place of linear approximations in pursuing the empirical objectives described throughout the book. Chapter 10 presents three leading alternatives to the linearization approach to model solution presented in chapter 2: projection methods, value-function iterations, and policy-function iterations. Chapter 11 then describes how the empirical methodologies presented in chapters 6–9 may be applied to nonlinear approximations of the underlying model produced by these alternative solution methodologies.

The key step in shifting from linear to nonlinear approximations involves the reliance upon simulations from the underlying model for characterizing its statistical implications. In conducting calibration and limited-information estimation analyses, simulations are used to construct numerical estimates of the statistical targets chosen for analysis, because analytical expressions for these targets are no longer available. And in conducting full-information analyses, simulations are used to construct numerical approximations of the likelihood function corresponding with the underlying model, using a numerical tool known as the particle filter.

The organization we have chosen for the book stems from our view that the coverage of empirical applications involving nonlinear model approximations is better understood once a solid understanding of the use of linear approximations has been gained. Moreover, linear approximations usefully serve as complementary inputs into the implementation of nonlinear approximations. However, if one wished to cover linear and nonlinear applications in concert, then we suggest the following approach. Begin exploring model-solution techniques by covering chapters 2 and 10 simultaneously. Then having worked through chapter 3 and sections 4.1 and 4.2 of chapter 4, cover section 4.3 of chapter 4 (the Kalman filter) along with section 11.2 of chapter 11 (the particle filter). Then proceed through chapters 5–9 as organized, coupling section 7.3.4 of chapter 7 with section 11.1 of chapter 11.

In the spirit of reducing barriers to entry into the field, we have developed a textbook Web site that contains the data sets that serve as examples

throughout the text, as well as computer code used to execute the methodologies we present. The code is in the form of procedures written in the GAUSS programming language. Instructions for executing the procedures are provided within the individual files. The Web site address is <http://www.pitt.edu/~dejong/text.htm>. References to procedures available at this site are provided throughout this book. In addition, a host of freeware is available throughout the Internet. In searching for code, good starting points include the collection housed by Christian Zimmerman in his Quantitative Macroeconomics Web page, and the collection of programs that comprise DYNARE:

<http://dge.repec.org/>

<http://www.cepremap.cnrs.fr/~michel/dynare/>

Much of the code provided at our Web site reflects the modification of code developed by others, and we have attempted to indicate this explicitly whenever possible. Beyond this attempt, we express our gratitude to the many generous programmers who have made their code available for public use.

### 1.3 Notation

A common set of notation is used throughout the text in presenting models and empirical methodologies. A summary is as follows. Steady state values of levels of variables are denoted with an upper bar. For example, the steady state value of the level of output  $y_t$  is denoted as  $\bar{y}$ . Logged deviations of variables from steady state values are denoted using tildes; e.g.,

$$\tilde{y}_t = \log\left(\frac{y_t}{\bar{y}}\right).$$

The vector  $x_t$  denotes the collection of model variables, written (unless indicated otherwise) in terms of logged deviations from steady state values; e.g.,

$$x_t = [\tilde{y}_t \quad \tilde{c}_t \quad \tilde{n}_t]'$$

The vector  $v_t$  denotes the collection of structural shocks incorporated in the model, and  $\eta_t$  denotes the collection of expectational errors associated with intertemporal optimality conditions. Finally, the  $k \times 1$  vector  $\mu$  denotes the collection of “deep” parameters associated with the structural model.

Log-linear approximations of structural models are represented as

$$Ax_{t+1} = Bx_t + Cv_{t+1} + D\eta_{t+1}, \quad (1.1)$$

where the elements of the matrices  $A$ ,  $B$ ,  $C$ , and  $D$  are functions of the structural parameters  $\mu$ . Solutions of (1.1) are expressed as

$$x_{t+1} = F(\mu)x_t + G(\mu)v_{t+1}. \quad (1.2)$$

In (1.2), certain variables in the vector  $x_t$  are unobservable, whereas others (or linear combinations of variables) are observable. Thus filtering methods such as the Kalman filter must be used to evaluate the system empirically. The Kalman filter requires an observer equation linking observables to unobservables. Observable variables are denoted by  $X_t$ , where

$$X_t = H(\mu)'x_t + u_t, \quad (1.3)$$

with

$$E(u_t u_t') = \Sigma_u.$$

The presence of  $u_t$  in (1.3) reflects the possibility that the observations of  $X_t$  are associated with measurement error. Finally, defining

$$e_{t+1} = G(\mu)v_{t+1},$$

the covariance matrix of  $e_{t+1}$  is given by

$$Q(\mu) = E(e_t e_t'). \quad (1.4)$$

Given assumptions regarding the stochastic nature of measurement errors and the structural shocks, (1.2)–(1.4) yield a log-likelihood function  $\log L(X|\Lambda)$ , where  $\Lambda$  collects the parameters in  $F(\mu)$ ,  $H(\mu)$ ,  $\Sigma_u$ , and  $Q(\mu)$ . Often, it will be convenient to take as granted mappings from  $\mu$  to  $F$ ,  $H$ ,  $\Sigma_u$ , and  $Q$ . In such cases the likelihood function will be written as  $L(X|\mu)$ .

Nonlinear approximations of structural models are represented using three equations, written with variables expressed in terms of levels. The first characterizes the evolution of the state variables  $s_t$  included in the model:

$$s_t = f(s_{t-1}, v_t), \quad (1.5)$$

where once again  $v_t$  denotes the collection of structural shocks incorporated in the model. The second equation is known as a policy function,

which represents the optimal specification of the control variables  $c_t$  included in the model as a function of the state variables:

$$c_t = c(s_t). \quad (1.6)$$

The third equation maps the full collection of model variables into the observables:

$$X_t = \tilde{g}(s_t, c_t, v_t, u_t) \quad (1.7)$$

$$\equiv g(s_t, u_t), \quad (1.8)$$

where once again  $u_t$  denotes measurement error. Parameters associated with  $f(s_{t-1}, v_t)$ ,  $c(s_t)$ , and  $g(s_t, u_t)$  are again obtained as mappings from  $\mu$ , thus their associated likelihood function is also written as  $L(X|\mu)$ .

The next chapter has two objectives. First, it outlines procedures for mapping nonlinear systems into (1.1). Next, it presents various solution methods for deriving (1.2), given (1.1).